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Hybrid Fuzzy Fed PID Control of Nonlinear Systems

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الملخص

التحكم المهجن الضبابي ذو التغذية الراجعة من نوع (بي أي دي) للأنظمة الغير خطية

تصميم أنظمة التحكم للأنظمة غير الخطية في الصناعة يعتبر مهمة معقدة. أحد الطرق التي أظهرت حلول لمشاكل الأنظمة غير الخطية هي استخدام التحكم المنطقي الـضبابي. هـذه الورقة تقترح طريقة جديدة لاستخدام التحكم التناسبي- التكاملي- التفاضلي (بي أي دي) وهـي أنظمة التحكم الضبابية الهجينة (بي أي دي) للأنظمة غير الخطية. السمة البارزة للنهج المقتـرح هو انه يجمع بين طريقة الكسب المجدولة الضبابية وبين التحكم المغذى (بي أي دي) لحل مشكلة التحكم غير الخطية. القواعد الناتجة للمتحكم المقترح تحتوي على قسم غير مثالي. لقد تم تقليـل عدد القواعد الضبابية باستخدام طريقة القواعد الضبابية التسلسلية. النتائج المأخوذة علـى جهـاز الحاسوب توضح أن كفاءة الطريقة الجديدة وهي طريقة التحكم الـضبابي المهجـن بـي أي دي المغذى تعطي أفضل نتائج من أي طريقة تقليدية خصوصا إذا اسـتخدمت طريقـة المامـداني، وبالتالي ستظهر أفضل نتائج مع تقليل عدد القواعد الضبابية.

ABSTRACT

Hybrid Fuzzy Fed PID Control of Nonlinear Systems

A new method of Proportional–Integral-Derivative (PID) controller is proposed in this paper for a hybrid fuzzy PID controller for nonlinear system. The important feature of the proposed approach is that it combines the fuzzy gain scheduling method and the fuzzy fed PID controller to solve the nonlinear control problem. The resultant fuzzy rule base of the proposed controller can be divided into two parts. In the upper part, the gain scheduling method is incorporated with a mamdani fuzzy logic controller to linearize the nonlinear system. In the lower part, a fuzzy fed PID controller is derived for all the locally linearized systems. The simulation results of a nonlinear system show that the performance of the fed PID hybrid fuzzy controller is better than that of the conventional fuzzy PID controller.

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Index Terms - Fuzzy gain scheduling, fuzzy fed proportional–integral-derivative (PID), nonlinear system.

I. INTRODUCTION

PID control is widely used in industrial applications although it is a simple control method. Stability of PID controller can be guaranteed theoretically, and zero steady-state tracking error can be achieved for linear plant in steady-state phase. Computer simulations of PID control algorithm have revealed that the tracking error is quite often oscillatory, however, with large amplitudes during the transient phase. To improve the performance of the PID controllers, several strategies have been proposed, such as adaptive and supervising techniques.

Fuzzy control methodology is considered as an effective method to deal with disturbances and uncertainties in terms of ambiguity. Fuzzy PID controller combining fuzzy technology with traditional PID control algorithm to become more effective in artificial intelligence control [1],[2].

The most common problem resulted early depending on the complexity of Fuzzy Logic Control (FLC) is the tuning problem. It is hard to design and tune FLCs manually for the most machine problems especially used in industries like nonlinear systems. For alleviation of difficulties in constructing the fuzzy rule base, there is the conventional nonlinear design method [3] which was inherited in the fuzzy control area, such as fuzzy sliding control, fuzzy scheduling [4],[5], and adaptive fuzzy control [6],[7]. The error signal for most control systems is available to the controller if the reference input is continuous. The analytical calculations present two-inputs FLC employing proportional error signal and velocity error signal. PID controller is the most common controller used in industries, most of development of fuzzy controllers revolve around fuzzy PID controllers to insure the existence of conventional controllers in the overall control structure, simply called Hybrid Fuzzy Controllers [8],[9].

The key idea of the proposed method is as follows: First, the fuzzy gain scheduling method is applied to linearize the nonlinear system at certain times. A fuzzy fed PID controller is designed by replacing the conventional PID controller with an incremental FLC. The Integral (I) part of the PID controller is fed by a differentiated feedback gain, this feedback for the integral part of PID controller is the new method used in this paper- we named it Fed PID- and it gives better results when compared with conventional PID control. Fuzzification of the reference input is performed for the system, while the control space of error signals is linearly partitioned after normalization. Fuzzy rule base is constructed recursively to obtain better nonlinear control as well as to guarantee closed-loop stability of the

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system. The proposed approach utilizes some modern control theorems, such as tuning in PID control. It can be emphasized that tuning the hybrid fuzzy controller is much easier than tuning a conventional fuzzy logic controller.

In section II, the fed PID Controller is proposed effectively. In Section III, the gain scheduling method is introduced as an effective nonlinear control method for nonlinear systems. In Section IV, a novel fuzzy fed PID controller is proposed. We show that recursive design of the fuzzy rule base can guarantee stability of local closed-loop systems. In Section V, control of a pole-balancing robot illustrates how the proposed design method can be easily applied to a nonlinear robotics system. Concluding remarks are given in the last section.

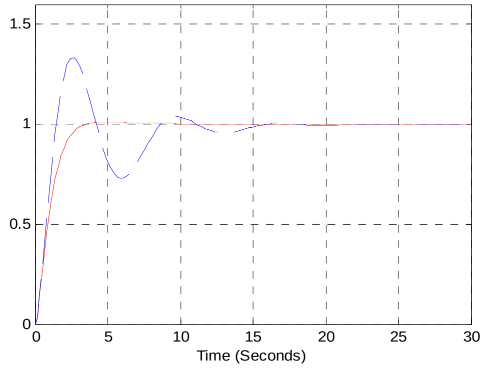
II. FED PID CONTROLLER

This type of PID controller is considered the contribution in this research. The name of fed is quoted from the feedback to the integrator (I) of the PID control and this feedback of the integrator is a differentiated feedback. This paper will show that fed PID control decreases the overshoot and the steady state error for PID controller design. One can ask why the differentiated feedback does not act on Proportional (P) or the Derivative (D) terms of the PID controller. Under experiments on the MATLAB simulation the step response shows only the positive response (minimum overshoot & steady state error) for the differentiated feedback that acts on the integrator.

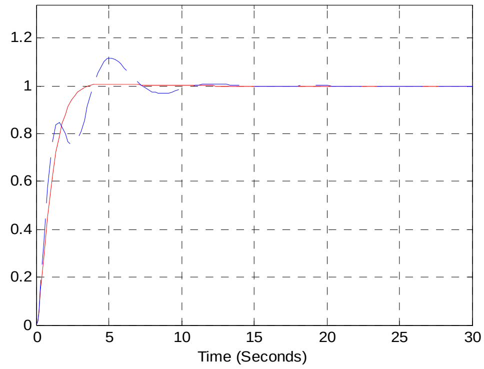
Figure 1.1 shows the step response of the proportional, derivative, and integral differentiated feedback versus the step response of the conventional PID controller for second order system. Note that the continuous line for the conventional PID controller. The mathematical description of the fed PID control is shown in equation (1),

$$T(s) = K_P + \left(\frac{K_I}{K_I + 1}\right)\frac{1}{s} + K_D s \qquad (1)$$

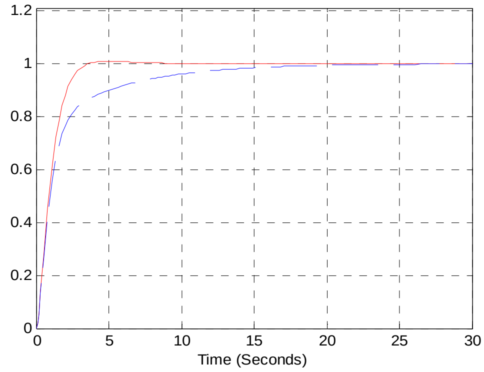
This description is derived from the block diagram shown in Figure 1.2 (b). This type of PID controller is used to decrease the steady state error and the overshoot as well.



(a) Differential P

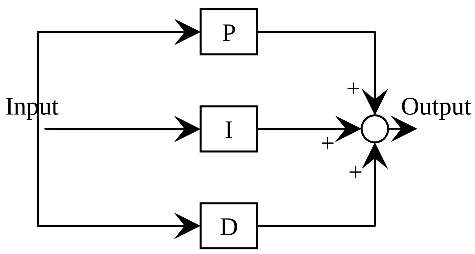


(b) Differential D

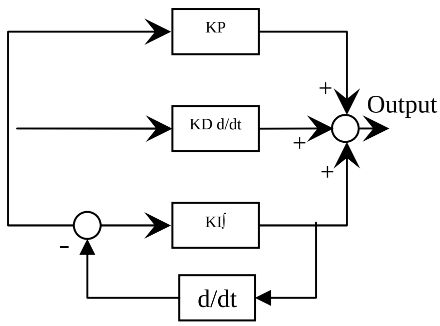


(c) Differential I

Figure 1.1: Step response of differential feedback (Fed) applied to (a) P, (b) D, (c) I.



(a) PID controller



(b) Fed PID

Figure 1.2: PID Controller versus Fed PID Controller.

Al-Aqsa University Journal (Natural Sciences Series), Vol.14, No.1, Pages 29-41, Jan.2010
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III. NONLINEAR CONTROL PROBLEM

Generally, most of robotics systems are nonlinear systems. One common task in robotics system control is to demand the robot or parts of the body to follow a given reference trajectory [10]. Tracking control of system dynamics may change significantly. Hence, instead of trying to model the system, a more feasible solution is to schedule the gains at each operating point. Since human expert can describes the system in a natural language better than mathematical equations, fuzzy control is also commonly used in nonlinear control of robotics systems [11],[12].

A. Gain Scheduling Method

Nonlinear systems can be generally expressed by the following nonlinear autonomous system equation:

$$\dot{x} = f(x) + g(x)u \tag{2}$$

Where $x = [x_1, x_2, ..., x_n]^T \in R^{n\times1}$ is the state vector, $u = [u_1, u_2, ..., u_m]^T \in R^{m\times1}$ is the control input vector, $f(x)$ and $g(x) \in R^{n\times1}$ are vector functions of states.

Assume $x^d(t) \in R^{n\times1}$ is the given reference trajectory whose corresponding reference input is $u^d(t)$

$$\overset{*}{x}{}^d = f(x^d) + g(x^d)u^d \tag{3}$$

Taking Lyapunov linearization around the operating points (x^d, u^d), we have

$$\dot{x} = \dot{x}^d + A(x^d)(x - x^d) + B(x^d)(u - u^d) \tag{4}$$

Where

$$A(x^d) = \left.\frac{df}{dx}\right|_{x=x^d} \quad B(x^d) = g(x^d) \tag{5}$$

Let $e = x - x^d, \dot{e} = \dot{x} - \dot{x}^d$ and (6)

System (4) is equivalent to

$$\dot{e} = A^d e + B^d u^e \tag{7}$$

where A^d and B^d are assumed to can be transformed into diagonal CCF, which is valid for many robotics systems. Because the reference trajectory x^d is a function of time, the nonlinear system (3) can be linearized at frozen time τ so that the tracking problem of the nonlinear system is transformed into a stabilization problem of the linear system (7) in the error state space. The equilibrium points are shifted from the reference trajectory points $x^d(\tau)$ to the origin. However, the aforementioned conventional gain-scheduling method employs linearization between two consecutive operating points. If the system states vary significantly along the time axis, global stability will be a problem. An alternative solution is to utilize fuzzy rules containing expert knowledge to perform smoother interpolation of all the operating points in the control envelope [13],[14].

B. Fuzzy Gain Scheduling

At some frozen times τ_i the corresponding control input can be approximated by (3), which is $x^d(\tau_i)$ or x^i shortly. In partitioning the state space, this x^i will be the centers of membership functions (MFs), LX^i [15]. The nonlinear system given by (3) can, therefore, be transformed into several local linearized systems

$$R^i : IF\ x^d\ is\ LX^i\ ,\ THEN\quad \dot{e} = A^i e + B^i u^e \qquad (8)$$

where A^i and B^i are system state matrices corresponding to x^i .

The control law to be designed is

$$R^i : IF\ x^d\ is\ LX^i\ ,\ THEN\quad u = u^d + u^e \qquad (9)$$

where u^d is the control input corresponding to the reference input x^d and u^e is the control input derived from error inputs.

The conventional approach of using the gain scheduling method is to design a linear controller for each local system in (8). The main advantage of this approach is that the powerful linear control theory may be applied. However, some simple nonlinear controllers like fuzzy PID controllers could be a better choice for handling the system nonlinearities. Then, the fuzzy PID controllers for local systems may be embedded in the global fuzzy gain scheduling rules to improve the integrity of the design, Moreover; the fuzzy fed PID controller will give the optimal solution more than any previous controller.

IV. HYBRID FUZZY CONTROLLER DESIGN

In this section, a fuzzy fed PID controller is proposed for enhanced control of the local linearized systems. By employing recursive feedback and appropriate tuning of conventional derivative gain, the fuzzy fed PID controller guarantees sector conditions of the output [10],[13]. Local stability analysis also explores the relationship between the conventional derivative gain and the fuzzy gain. Although the proposed controller is developed as a hybrid fuzzy fed PID controller, the overall structure shows its potential to be a new form of stand alone fed FLC depicted in Figure 2.

Al-Aqsa University Journal (Natural Sciences Series), Vol.14, No.1, Pages 29-41, Jan.2010
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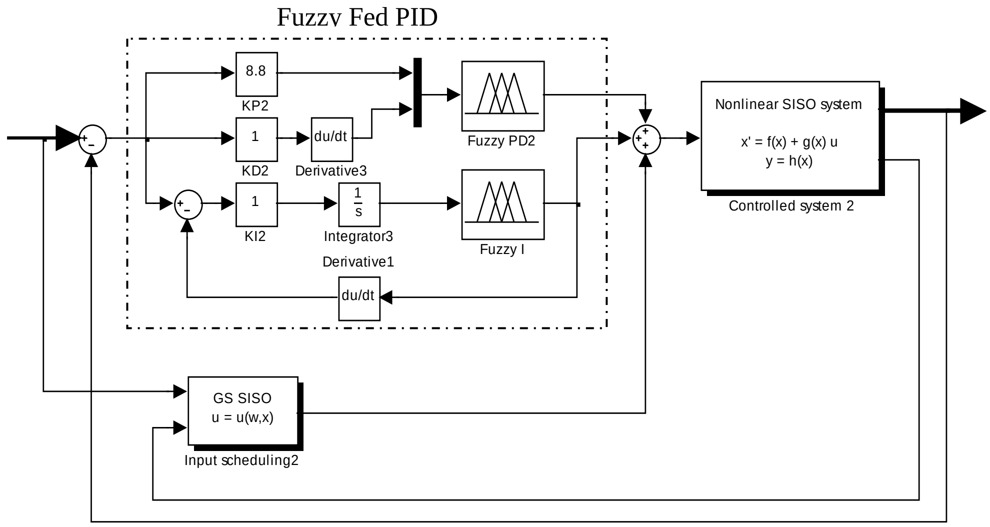


Figure 2: Overall Control

A fuzzy PID controller with fuzzy PI plus conventional derivative controller was proposed in [6] by discretizing the conventional PI controller and constructing from simple linear fuzzy rules in an incremental way. But in this section, a new type of fuzzy PID controller is proposed based on fuzzy fed PID control structure.

The fuzzy fed PID controller is constructed in an incremental way by employing both error signals and recursive feedback signals as inputs to fed PID the main idea is found in the integral side where the integral side when fed by a deferential feedback gives us a null overshoot and steady state error, the enhancement is very significant using fuzzy fed PID controller. The most widely adopted conventional PID controller structure used in industrial applications is the following structure [10]:

$$u_{PID}(t) = K_P^C e_v(t) + K_I^C e_p(t) + K_D^C e_a(t) \qquad (10)$$

where K_P, K_I, and K_D are the conventional proportional, integral, and derivative gains, respectively, and $u_{PID}(t)$ is the controller output and $e_v(t)$ is the velocity error signal, $e_p(t)=\int e_v(t)$ is the proportional error signal and $e_a(t)=de_v(t)/dt$ is the acceleration error signal.

The items in (10) form the PID control and can be replaced by the following linear fuzzy rules:

$$R^j : IF\ e_p\ is\ LE_p^j\ AND\ e_v\ is\ LE_v^j\ , THEN\ u_{PI}\ is\ LU_{PI}^j \qquad (11)$$

Where $LE_p^j\ and\ LE_v^j$ are the linguistic values of error signals of the j^{th} fuzzy rule and LU_{PI}^j is the linguistic value of the output $u_{PI}(t)$

The approximate expression of fed PID shown in equation (12), because the value of K_I is approximately 1 referring to Ziegler-Nichols method that calculate the value of PID gains :

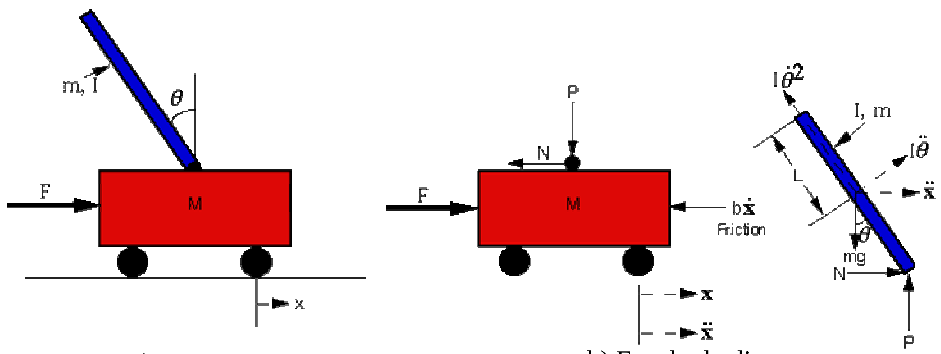
$$u_{PID}(t) = K_P^C e_v(t) + (0.5) K_I^C e_p(t) + K_D^C e_a(t) \qquad (12)$$

But the real output is differ when the fed PID controller is used where the fed PID controller has overshoot and steady state error less than the conventional PID controller.

Note that the output feedback from the integrator is taken from the output of the defuzzification process which gives the best results showing in the illustrative example.

V. ILLUSTRATIVE EXAMPLE

In the example, the proposed controller is used to control with an inverted pendulum robot, that robot is used in the most of our applications because of nonlinearity problem and marginally stability. Figure 3 shows the inverted pendulum and its free body diagram.



a) Structure b) Free body diagram

Figure 3: Inverted pendulum a) Structure b) Free body diagram

Al-Aqsa University Journal (Natural Sciences Series), Vol.14, No.1, Pages 29-41, Jan.2010
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The dynamic equation of the inverted pendulum robot which can be extracted from the free body diagram is given by [16]:

$$\ddot{\theta} = \frac{(m_p + m_c) g \sin\theta - m_p \dot{\theta}^2 \, l \sin\theta - F\cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \tag{13}$$

Where θ is the angle between the pendulum and the vertical, the angular velocity is expressed by $\dot{\theta}$, the force which acts on the cart is F, the gravity acceleration g is 9.8m/sec^2 , mc and mp are the mass of cart and the mass of pole respectively, and l is the half length of the pendulum. The system equation is written as follow [17]:

$$\dot{x} = f(x) + g(x)u \tag{14}$$

Where

$$f(x) = \begin{bmatrix} \dot{\theta} \\ \dfrac{(m_p + m_c) g \sin\theta - m_p \dot{\theta}^2 \, l \sin\theta \cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ \dfrac{-F\cos\theta}{(m_p + m_c) l (4/3 - m_p \cos^2\theta)} \end{bmatrix} \tag{15}$$

The last two equations are used for simulation without a previous technique of linearization because the fuzzy PID controller uses the linguistic formulas and makes a linearization of the nonlinear system. Beside the point that the amount of masses and measurements of the pendulum, the most point to be focused is the fed fuzzy PID controller is make lower overshoot and minimum steady state error.

Theoretically, the number of rules that cover all possible input variations for a five term fuzzy controller is (n1×n2×n3×n4×n5). Where (n1× n2× n3× n4× n5) are the number of membership functions or linguistic labels of the five input variables. In a particular case, if n1=n2=n3=n4=n5=5, then the number of rules will be 3125. In practical applications, the implementation of such a large rule base will take a lot of reasoning time besides a large amount of process memory [18].

The 27 rules used in the fuzzy PID can be reduced to 12 rules because the fuzzy PID controller is divided into two main rules the PD rules and the Integrator rules, I divide the integrator rules to make a feedback from the output of the integrator with a deferential feedback to the input of the integrator, this technique makes better results shown in Figure 5, the fuzzy rules of the fed PID controller shown bellow:

For the fuzzy proportional differentiator:

IF (e_v is –ve) AND (e_a is –ve) THEN (u_{PD} is –ve)
IF (e_v is –ve) AND (e_a is zero) THEN (u_{PD} is –ve)
IF (e_v is –ve) AND (e_a is +ve) THEN (u_{PD} is zero)
IF (e_v is zero) AND (e_a is –ve) THEN (u_{PD} is –ve)
IF (e_v is zero) AND (e_a is zero) THEN (u_{PD} is zero)
IF (e_v is zero) AND (e_a is +ve) THEN (u_{PD} is +ve)
IF (e_v is +ve) AND (e_a is –ve) THEN (u_{PD} is zero)
IF (e_v is +ve) AND (e_a is zero) THEN (u_{PD} is +ve)
IF (e_v is +ve) AND (e_a is +ve) THEN (u_{PD} is +ve)

For the fuzzy fed integrator:

IF (e_p is –ve) THEN (u_I is –ve)
IF (e_p is zero) THEN (u_I is zero)
IF (e_p is +ve) THEN (u_I is +ve)

Figure 4 illustrates the membership functions of the inputs to the controller and the output desired:

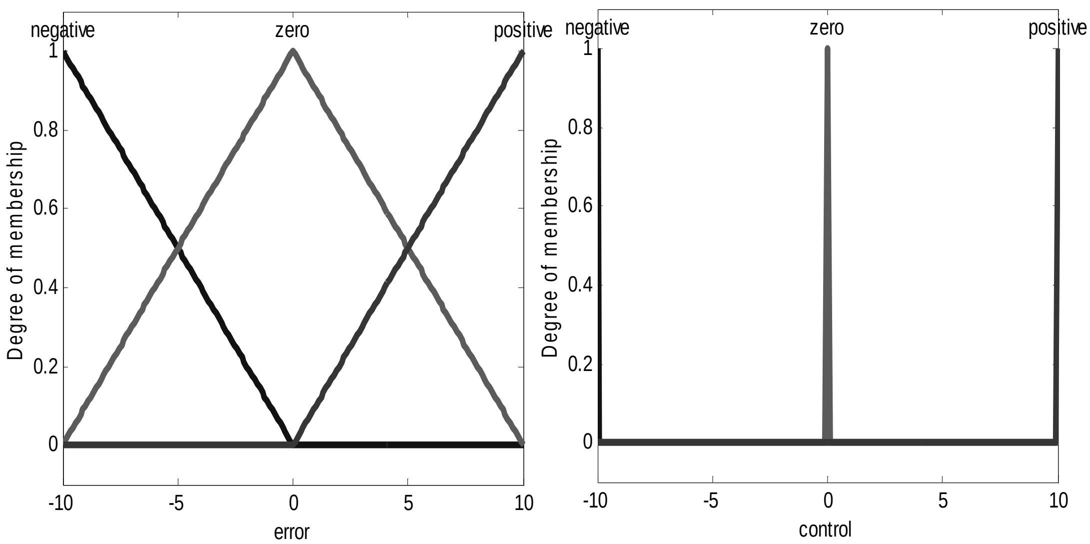

Figure 4: membership functions of the inputs and output

The fuzzy rules are extracted from an expert system and logical relations. In Figure 4 the membership functions for the error signal (*e_v, e_a, or e_p*) are limited from -10 to 10 degrees because the angle of the inverted pendulum will make the system unstable for more than 10 degrees. Also in Figure 4 the membership function of the control signal (u_{PID}) with value between -0.01 and 0.01 to minimize value of the angle to the smallest possible value.

Figure 5 illustrates the conventional PID versus fed PID of where the fed PID achieves an overshoot less than conventional PID controller, in addition the fed PID has zero steady state error but the conventional has some errors in the steady state. Table 1 shows the comparison between the conventional PID and the fed PID control of the nonlinear system.

Table 1: SSE and maximum OS of various control approaches.

Desired Control Approach	***OS%***	***SSE***
PD+I Conventional Control	5.12	1.53E-3
PI+D Conventional Control	4.23	1.20E-3
Fed PID Control	2.47	0.00

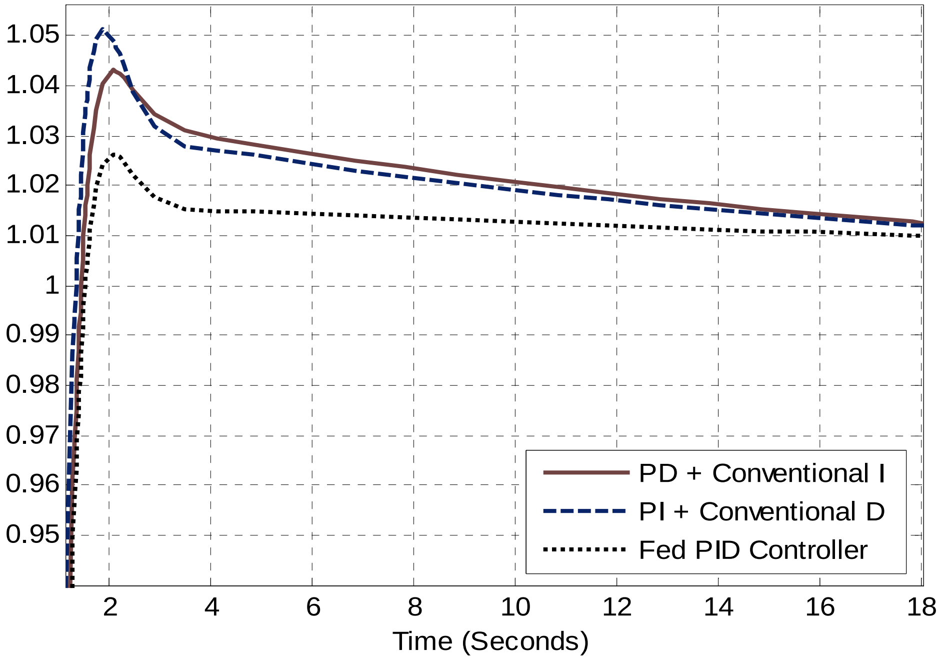

Figure 5: Stabilization control of the fed PID controller versus several controllers

VI. CONCLUSION

In this paper, the new approach of control design of a hybrid fuzzy PID controller is proposed. The proposed design method can construct the PID control using fuzzy rules without referring to numerical calculations. The proposed controller demonstrates excellent control performance for nonlinear robot which depends on the hybridizing of the gain scheduling method and fed PID controller which gives better control specifications towards the conventional PID, fuzzy PID and hybrid fuzzy PID . The proposed problem is one of the useful topics in the area of fuzzy control field related with robotics systems.

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