Applications of Metamaterials in Optical Waveguide Isolator

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الملخص

عازلات الدليل الموجي البصرية هي وحدات بصرية جوهرية متكاملة في أنظمة اتصال الألياف الضوئية المتقدمة. تعرض هذه الدراسة عازل بصري متكامل له تركيب بسيط يتكون من ثلاث طبقات: هي شريحة رقيقة من العقيق المغناطيسي محصورة بين الغطاء العازل الخطي وركيزة الميتامتريل (MTM). إن معامل الانكسار الفعال لكلتا الحقول الأمامية والخلفية قد حسب بشكل تحليلي باشتقاق معادلة تشتت المجالات المغناطيسية المستعرضة (TM). أما الاختلاف $(\Delta\beta)$بين المرحلة الثابتة للانتشار أماما وخلفا فقد حسب بشكل عددي للقيم المختلفة لمعامل السماحية (ε_s) والنفاذية (μ_s) لمادة الركيزة المصنوعة من الميتامتريل. وقد تم رسم $\Delta\beta$ بدلالة سمك الشريحة لقيم مختلفة لكل من μ_s و ε_s. وقد أوضحت النتائج أن قيمة $\Delta\beta$ تتغير بتغيير ثوابت الميتامتريل وتقل مع زيادة سمك الشريحة. كما أن القيمة القصوى $\Delta\beta_{max}$ تقل مع نقص الفرق بين معامل الركيزة وشريحة العقيق المغناطيسي وتصل الي أقل قيمة لها عندما تكون قيمة ε_s تساوي -0.5 نتائج هذه الدراسة تساعد المصممون في اختيار التصميم المثالي للعازل عندما تؤول $\Delta\beta$ إلى الصفر.

ABSTRACT

Optical waveguide isolators are vital integrated optic modules in advanced optical fiber communication systems. This study demonstrates an integrated optical isolator which has simple structure consisting of three layers. A thin magnetic garnet film is sandwiched between linear dielectric cover and metamaterial (MTM) substrate. The effective refractive indexes for both forward and backward fields were analytically calculated by deriving the dispersion equation of the transverse magnetic fields (TM). The

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difference ($\Delta\beta$) between the phase constant for the forward and backward propagations was calculated numerically for different values of MTMs permittivity (ε_s) and permeability (μ_s). $\Delta\beta$ is plotted as a function of the film thickness for different values of ε_s and μ_s. Results show that the value of $\Delta\beta$ changes with the parameters of MTMs and decreases as the film thickness increases. Results also show that $\Delta\beta_{max}$ decreases with decreasing the step difference between the film and the substrate. Moreover, the lowest value of $\Delta\beta_{max}$ occurs at ε_s = -0.5. The findings of this study help designers in selecting the optimal design for the isolator at which $\Delta\beta$ approaches zero.

I. Introduction:

Optical communication via glass fiber is an attractive technique for high data rate transmission. Thus, it is important that the semiconductor lasers as a reliable light source be protected from reflected light, otherwise they become unstable. For this purpose, optical isolators, which depend on the nonreciprocal Faraday rotation of magnetooptical materials are needed. Such nonreciprocal devices are commercially available as micro-optical components (Chen and Kumarswami, 1986). The advantages of optical isolator include compatibility with waveguide optics, low magnetic field requirements, and low cost (Shintaku, 1998).

An integration of optical devices is expected to play an important role in laser communication systems. To fully implement all functions in the optical signal processing, nonreciprocal devices such as isolators are needed (Sun et al., 1977). Therefore, it is crucial to integrate optical components like lasers, isolators, circulators, modulators combiners, splitters, and couplers on a single substrate. Light is guided in such optical circuits by monomode dielectric waveguides, which have lateral dimensions in the range of micrometers. As many devices depend on interference effects, the geometrical tolerances of the waveguides are very essential.

A sever integration problem arise if materials with large refractive index difference must be combined, like semiconductors (laser materials) and garnets. Magnetic Garnet films are promising materials for magneto-optic applications. These films can be used as an optical isolator, as a switch for printer, and as a switch for a display (Hibiya, 1985). In the near infrared region, where an optical glass fiber transmission is developed, magnetic garnet films can be used as optical waveguides. They combine high Faraday rotation with low optical losses; therefore, they are highly qualified for integrated optical devices (Dötsch, 1992).

Optical nonreciprocal devices are indispensable for the protection of active devices from unwanted reflected light. The nonreciprocal phase shift occurs in transverse magnetic modes (TM)[1] that travel in magneto-optic waveguides in which magnetization is aligned transverse to the light propagation direction in the film plane (Yokoi, 2000).

Various attempts have been made to build optical waveguide isolators of different types (Wolfe et al., 1990; Yamamoto and Makimoto, 1974; Ando et al., 1988; Pross et al., 1988; Hernhndez et al., 1988). Waveguide optical

[1] TM modes have no magnetic field in the direction of propagation and TE modes have no electric field in the direction of propagation (Reitz et al., 1993).

isolators have been intensively investigated for many years. Numerous concepts were developed and studied theoretically and experimentally, where different kinds of nonreciprocal effects are exploited. Some promising concepts of integrated optical isolators rely on the nonreciprocal phase shift of TM modes, which is the difference $\Delta\beta = \beta^{+} - \beta^{-}$ between the forward and backward propagation constants of TM modes (Damman et al., 1990). $\Delta\beta$ depends on the waveguide structure, the misfit between the lattice constant of the film and the substrate, and very much on the growth conditions for the epitaxial layers. The absolute value of $\Delta\beta$ can be optimized by choosing a proper geometry of the waveguides (Damman et al., 1990).

In an isolator, the light passing the waveguide in the reverse direction must be blocked by the first polarizer. Naturally, this blocking can be perfect for elliptically polarized light, thus yielding too low values for the degree of isolation for waveguides with $\Delta\beta \neq 0$. It is very difficult to make magneto-optic waveguide in which $\Delta\beta$ is sufficiently small. The thickness at which the difference $\Delta\beta$ reaches maximum guides only zero modes of transverse magnetic and electric fields TM_0 and TE_0[2] (Dötsch, 1992).

The dielectric permittivity ε and the magnetic permeability μ are the only substance parameters that appear in the dispersion equation; therefore, they are the characteristic quantities which determine the propagation of electromagnetic waves in matter. Veselago (1968) has predicted several new electrodynamics phenomena of substances with simultaneously negative values of ε and μ. These substances are named metamaterials (MTMs) and also called double-negative media (Veselago, 1968, Pendry, 1999).

Due to the double-negative property of MTMs, the electromagnetic waves propagations along these media are quite unusual. Surface waves at the interface between the double negative media and dielectric media vary in their propagation behaviors from those at the interface between two unlike conventional dielectric media. For example, slab waveguides with double-negative media also have extraordinary guided dispersion characteristics (Kim et al., 2006). Stress effect on the performance of optical waveguide sensor consists of dielectric slab inserted between metamaterial cladding and substrate has been investigated by using numerical calculations (El-Khozondar et al., 2008).

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[2] TM_0 and TE_0 are the zero order modes of the TM and TE modes (Reitz et al., 1993).

The main purpose of this paper is to present the application of MTMs in the structure of the optical waveguides to realize integrated nonreciprocal devices like isolators. The role of the MTMs parameters on the guided dispersion characteristic of the waveguide isolator is investigated by introducing several sets of negative permittivity and permeability with their products kept the same. Findings associated with the effect of the material constants were observed and discussed in section V, based upon the qualitative description of the MTM waveguide isolator in section II. Theoretical analysis of magneto-optic waveguides is described in section III. Section IV is dedicated to the calculation of modal field profiles.

II. Waveguide Isolator Configuration

As shown in Figure 1, the basic waveguide arrangement in the present analysis consists of three layers: the cover, the film, and the substrate. The cover region is filled with linear dielectric material and the substrate region is filled with metamaterials. The substrate and the cover region are isotropic, having dielectric constants ε_s and ε_c respectively. The dielectric tensor ε is defined for the iron garnet film as (Doormann et al., 1984)

$$\varepsilon = \begin{pmatrix} \varepsilon_{xx} & 0 & i\varepsilon_{xz} \\ 0 & \varepsilon_{yy} & 0 \\ -i\varepsilon_{xz} & 0 & \varepsilon_{zz} \end{pmatrix} \qquad (1)$$

assuming that the magnetization $\vec{M}$ is adjusted in the film plane perpendicular to propagation. As we consider loss free materials, all numbers, ε_{ij} are real. Moreover, all diagonal elements are equal, e.g. $\varepsilon_{xx}= \varepsilon_{yy}= \varepsilon_{zz}$ based upon the isotropic property of the magneto-optic materials (Boardman and Xie, 1997). The magnetooptical properties are described by the off-diagonal components of the permittivity tensor, $i\varepsilon_{xz}$, where ε_{xz} is related to the specific Faraday rotation θ_F by $|\varepsilon_{xz}| \approx 2n_{film}|\theta_F|/k$, where n_{film} is the refractive index of the film and $k=2\pi/\lambda$ the wave number in vacuum with λ the wavelength in vacuum.

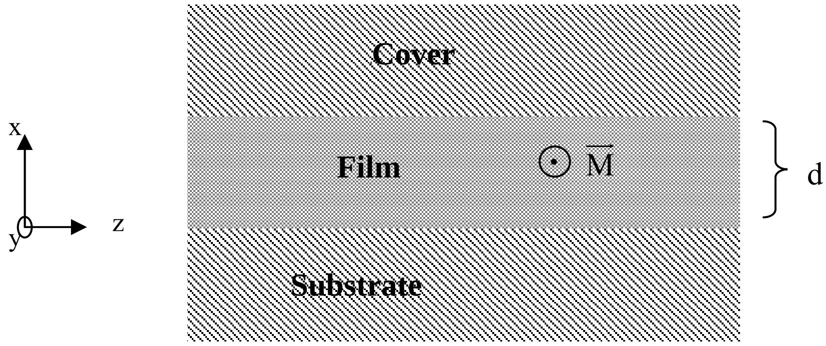


Figure 1: Basic Geometry of the waveguide isolator.

III. Theoretical Method

The basic principle of an isolator is that the incoming light is linearly polarized by the first polarizer. The plane of polarization is rotated at 45° by the Faraday rotator. Then, the light passes the second polarizer, set at 45° with respect to the first one. The plane of polarization of reflecting light traversing the Faraday rotation in backward direction is rotated by another 45° and thus the reflected light is blocked by the first polarizer.

The theoretical analysis of magneto-optic waveguides is accomplished as follows. The modal field equations are derived for each layer by applying the electric and magnetic fields of the propagating modes into Maxwell's equations. Then, boundary conditions are applied at the interfaces between the three layers. This produces the transverse equation which is solved numerically to obtain the difference between forward and backward propagation in terms of distance. The magnetooptic effect lead to nonreciprocal mode conversion and nonreciprocal phase shifts. Finally these effects are used to design nonreciprocal devices as isolators.

IV.Calculation of Modal Field Profiles

It is taken that the waveguide is homogeneous in the z-direction, the direction of light propagation, and that all the materials are lossless. In this work we will consider only TM modes. Thus, the propagating modes are described by their electric and magnetic fields, respectively:

$$\vec{E} = \left(E_x, 0, E_z\right) \exp\left[i\left(\omega t\text{-}\beta z\right)\right],$$
$$\vec{H} = \left(0, H_y, 0\right) \quad \exp\left[i\left(\omega t\text{-}\beta z\right)\right], \tag{2}$$

where $\beta = n_{eff}k$ denotes the propagation constant, n_{eff} the effective refractive index, $\omega = ck$ the frequency, and c is the speed of light. In the film region, Maxwell's equations yield for the transverse components of the electric and magnetic field vectors the differential equations:

$$\frac{dE_z}{dx} + i\beta E_x = i\mu\omega H_y, \tag{3}$$

$$\frac{dH_y}{dx} = \omega\varepsilon_{xz}E_x + i\varepsilon_{xx}E_z\omega, \tag{4}$$

$$H_y = \frac{\omega\varepsilon_{xx}}{\beta} E_x + \frac{i\varepsilon_{xz}\omega}{\beta} E_z, \tag{5}$$

$$E_z = \frac{1}{i\varepsilon'\omega}\left[\frac{dH_y}{dx} - \frac{\beta\varepsilon_{xz}}{\varepsilon_{xx}} H_y\right], \tag{6}$$

where the effective dielectric constant is given by $\varepsilon' = \varepsilon_{xx} - \frac{\varepsilon_{xz}^2}{\varepsilon_{xx}}$. Solving the differential equations (2), (3), (4), (5), and (6) gives the differential equation:

$$\frac{d^2H_y}{dx^2} + (k^2\varepsilon' - \beta^2)H_y = 0. \tag{7}$$

The solution of the differential equation (7) is,

$$H_y = B\,sin(\,px) + C\,cos(\,px), \tag{8}$$

where $p = k\sqrt{\varepsilon' - n_{eff}^2}$, and B and C are constants. For the film, the corresponding electric field component, E_z is defined by equation (6).

In the cover and substrate region, Maxwell's equations yield for the transverse components of the electric and magnetic field vectors the differential equation:

$$\frac{d^2H_y}{dx^2} + (\beta^2 - k^2n_i^2)H_y = 0, \tag{9}$$

where $n_i = \varepsilon_i \mu_i$, i denotes c for cover and s for substrate. In the core region, the solution of the differential equation (9) is,

$$H_y = A\, e^{-q_c x}, \tag{10}$$

where $q_c = k\sqrt{n_{eff}^2 - n_c^2}$, and A is constant.

In the substrate region, the solution of the differential equation (9) is,

$$H_y = D\, e^{q_s x}, \tag{11}$$

where $q_s = k\sqrt{n_{eff}^2 - n_s^2}$, and D is constant. For the cover and the substrate, the corresponding electric field component, E_z is

$$E_z = \frac{1}{i\omega\varepsilon}\,\frac{dH_y}{dx}. \tag{12}$$

Using the solutions for H_y and E_z and applying the boundary conditions (continuity of E_z and H_y), we obtain the transverse equation:

$$pd = tan^{-1}\left(\frac{\varepsilon_{eff}q_c}{p\varepsilon_c} - \frac{\varepsilon_{xz}\beta}{\varepsilon_{xx}p}\right) + tan^{-1}\left(\frac{\varepsilon_{eff}q_s}{p\varepsilon_s} - \frac{\varepsilon_{xz}\beta}{\varepsilon_{xx}p}\right). \tag{13}$$

Equation (13) clearly shows that there are nonreciprocal solutions because of the linear terms in β. Therefore, its solution depends on the direction and has different values for forward propagation (β^+) and backward propagation (β^-). For this reason, such a waveguide acts as a nonreciprocal phase shifter. The transverse equation (equation 13) is solved numerically for β^+ and β^-. The difference between forward and backward propagation ($\Delta\beta = \beta^+ - \beta^-$) is numerically calculated in terms of the film thickness (d). The relation between Δβ and the refractive index step between waveguide film and substrate is also calculated numerically.

The effect of the MTM parameters on the guided dispersion characteristic of the waveguide isolator is investigated by introducing several sets of negative permittivity and permeability (ε_s and μ_s) with their products kept the same, *i.e.*, $\varepsilon_s\mu_s = 4.0$ (K. Y. Kim *et al.*, 2006). This value is chosen such that the film will maintain higher refractive index than the surroundings. The metamaterial parameters in the substrate region are chosen as listed in Table 1. The smallest value for ε_s is taken to be -0.5 following the previous works using metamaterials (K. Y. Kim *et al.*, 2006). The calculation of $\Delta\beta$ as a function of d is repeated for each selected set of ε_s and μ_s.

The parameters used in the present calculations are: $\varepsilon_c = 1.0$, $\varepsilon_{xx} = 5.3$, $\varepsilon_{xz} = \pm 0.005$ where (+) for forward propagations and (–) for backward propagations and $k = 4.83\ \mu m^{-1}$. The value of $\varepsilon_{xx} = 5.3$ considers the refractive index variations induced by materials mismatch substitution. The value $\varepsilon_{xz} = 0.005$ matches a specific Faraday rotation of 2880º/cm at a wavelength of 1.3 μm (Hibiya, 1985).

Table 1: Combinations of ε_s and μ_s of MTMs in the present study. The product values are kept constant, *i.e.*, $\varepsilon_s\mu_s = 4.0$ (K. Y. Kim *et al.*, 2006).

	ε_s	μ_s
1.	-8.0	-0.5
2.	-4.0	-1.0
3.	-2.0	-2.0
4.	-1.0	-4.0
5.	-0.8	-5.0
6.	-0.5	-8.0

V. Results

Figure 2 demonstrates the relation between phase difference $\Delta\beta$ and the film thickness d for different values of MTMs parameters ε_s and μ_s as indicated in table 1. From figure 2, it is clear that the value of $\Delta\beta$ depends on both the core thickness (d) and the substrate parameters ε_s and μ_s. The value of $\Delta\beta$ decreases as d increase and approaches zero at d=0.25 μm. It also shows that the change of ε_s causes variation of $\Delta\beta_{max}$. In the figure, we can see that the value of $\Delta\beta_{max}$ decreases as the value of ε_s decreases. To optimize the isolator behavior, the value of $\Delta\beta_{max}$ should approach zero, which in our proposed structure occurs at less negative values of ε_s and at very small values of d. Similar results have been observed by Dötsch e. al.

where they used a similar structure with linear cover and substrate (Dötsch e. al., 1992).

Figure 3 shows the relation between the value of $\Delta\beta_{max}$ and the refractive index step between waveguide film and substrate (ε_{xx}- ε_s). In all calculations, the value of ε_{xx}=5.3 is kept constant to emphasize the effect of MTMS parameters (ε_s, μ_s). Figure 3 exhibits that as the refractive index step (ε_{xx}- ε_s) decreases, the value of $\Delta\beta_{max}$ decrease and approaches its lowest value at the lowest chosen value of ε_s = -0.5. Lower values of ε_s such as -0.3 (μ_s= 40/3) and -0.1 (μ_s= 40) are not shown in the figure because the values of $\Delta\beta_{max}$ are highly fluctuated with the thickness. Thus, the less negative values of ε_s are not recommended to improve the efficiency of the isolator. This is consistent with previous works (K. Y. Kim *et al.*, 2006).

The optimized isolator behavior occurs at $\Delta\beta_{max}$ equals zero. The results show that we can control the behavior of the optical isolator by varying the values of MTMS parameters (ε_s, μ_s) and adjusting the film thickness according to that.

VI.Conclusion

The utilization of metamaterials substrate in the structure of three-layer waveguide isolator has been presented. The guided dispersion characteristics of the optical waveguide isolator are numerically investigated for different values of the MTM substrate parameters. All calculations are done with the product of the permittivity and permeability of the MTM substrate kept constant at $\varepsilon_s\mu_s$ = 4.0. The results show that at the value of $\Delta\beta$ approaches very close to zero at d=0.25 µm. The results also indicate that the minimum value of $\Delta\beta_{max}$ occurs at the lowest value of ε_s = -0.5. Since the important parameter in designing optical isolator is to have small value of $\Delta\beta_{max,}$ the results are encouraging to propose an optical isolator.

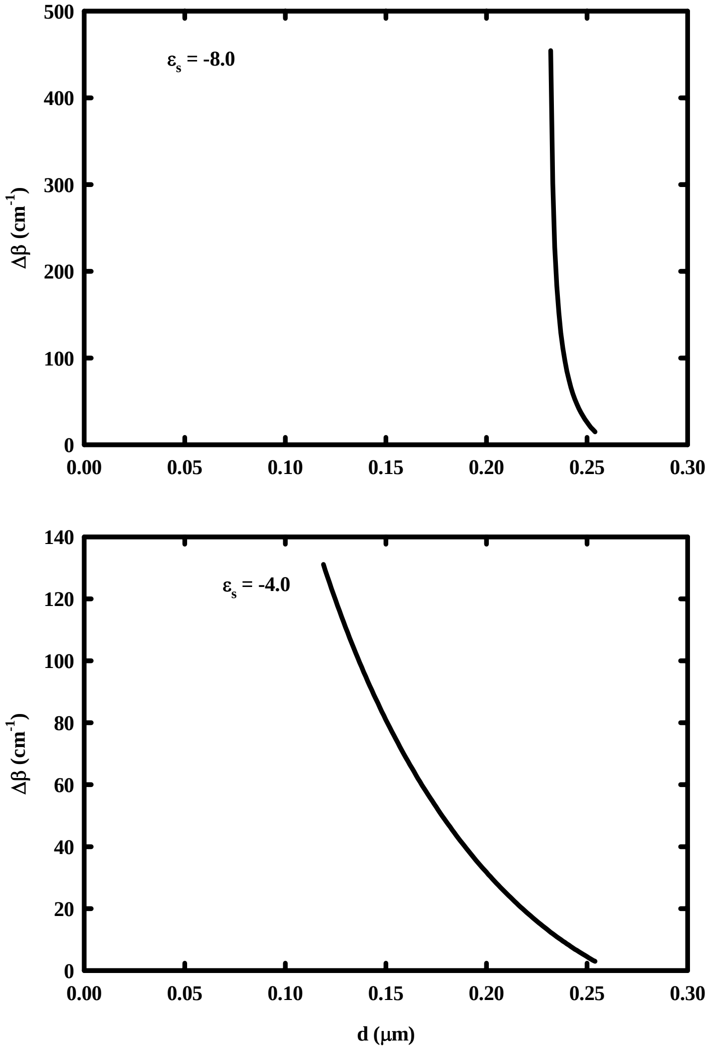


Figure 2: The difference $\Delta\beta$ between the phase constants for the forward and backward propagation as a function of the film thickness d at different values of substrate permittivity ε_s. The value of ε_s is indicated. The product values of ε_s and μ_s are kept, i.e., invariant $\varepsilon_s\mu_s = 4.0$.

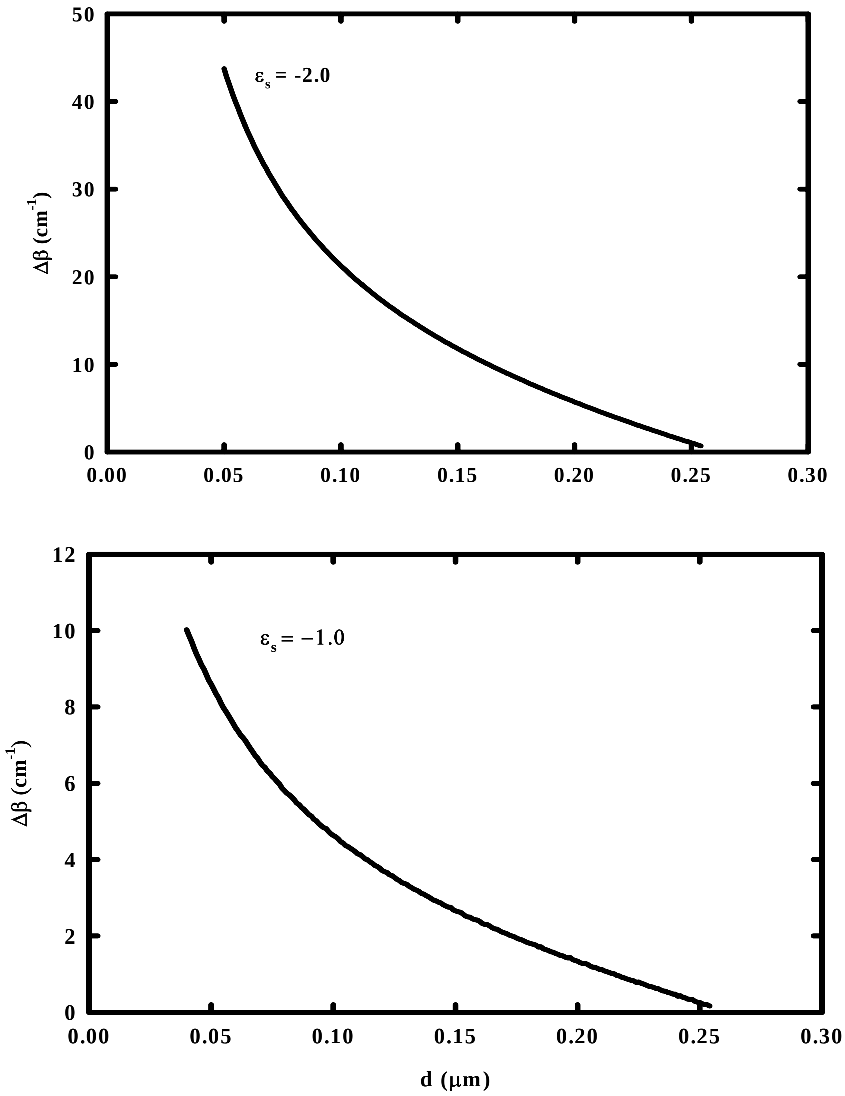
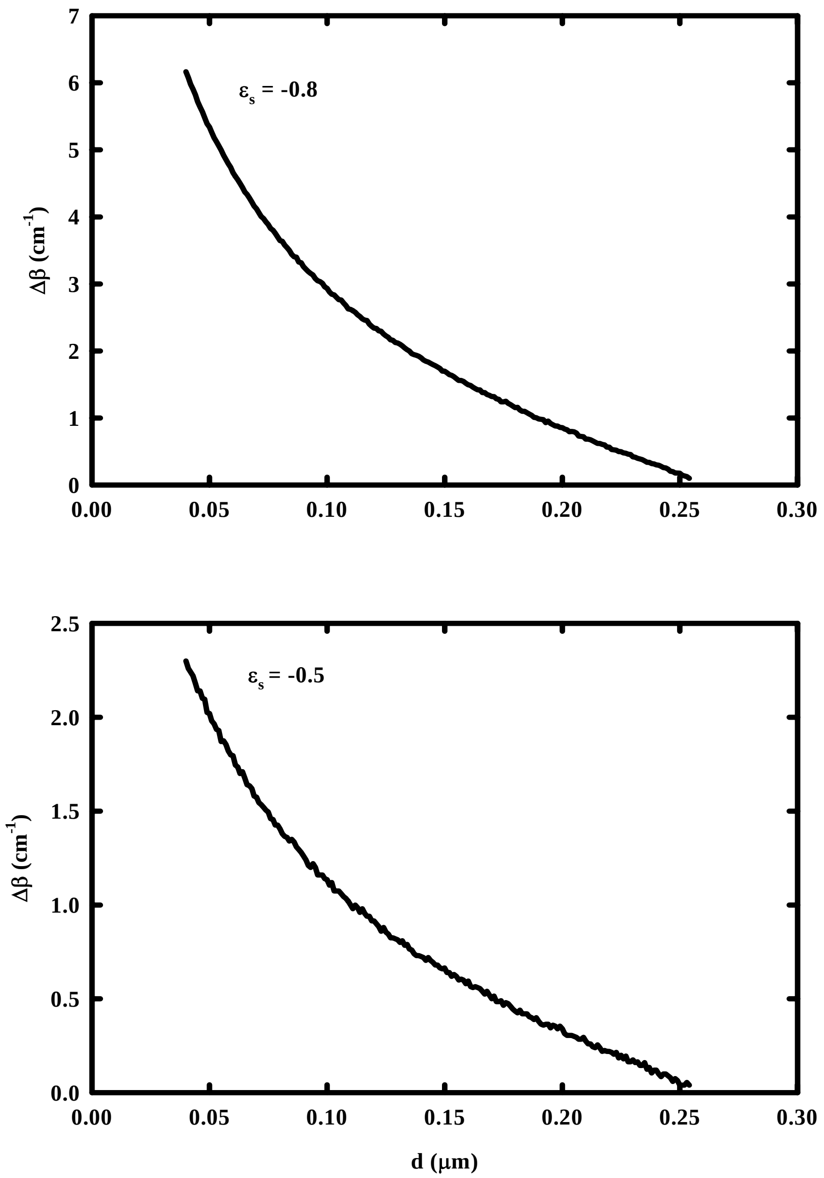


Figure 2: Continued



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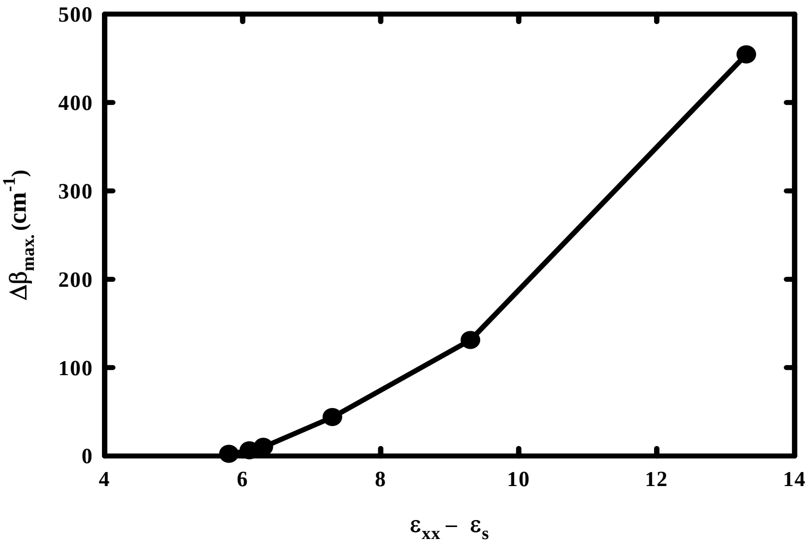

Figure 3: $\Delta\beta_{max}$ dependence of the difference ε_{xx}-ε_s. The value of ε_{xx}=5.3.

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