AL-AQSA UNIVERSITY
Department of mathematics
Date: 3/6/2018

جامعة الأقصى

Topology (**Math 4360**)
Final Exam
Time: Two Hours

الرقم الجامعي:

الاسم:

Answer all of the following questions:

Q1) a)(**5 pt.)** Show that (X,τ) is T_1 if and only if each point $x \in X$ is a closed subset of X.

b) **(5 pt.)** Show that int(A) $\cup$ int (B) $\subseteq$ int (A $\cup$ B) . Show that the equality may not always hold

Q2) a) Which of the following topologies are homeomorphic, explain your answer

1) **(3 pt.)** R and R^2 with the standard topology on each.

2) **(3 pt.)** the left ray and the cofinite topology on R.

3) **(3 pt.)** (0,1) and R with the standard topology on each.

4) **(3 pt.)** Z and N (where Z is the set of integer and N is the set of natural number)

b) **(6 pt.)** Show that $\overline{A_1 \times A_2} = \overline{A_1} \times \overline{A_2}$

c) **(6 pt.)** For R^2 with the product topology induced by the base $R_{standard} \times R_{standard}$.

Let **A**= **N× (0, ∞)** , **B={ (x,y) ⊆R²: x-y=1}** find

Int(A)=	Bd(A)=	A'=	$\overline{A} =$
Int(B)=	Bd(B)=	B'=	$\overline{B} =$

Q3) a) **(5 pt.)** Show that $f:(R, R_{standard}) \to (R, R_{standard})$ defined by $f(x) = x^2$ is continuous.

b) Give example to show that the following is not hold:

1) **(2 pt.)** T_0 –space → T_1 –space.

2) **(2 pt.)** Regular → T_2 –space.

3) **(2 pt.)** Every injection function is continuous.

c) (4 pt.) Show taht T_4 –space $\rightarrow$ T_3 -space.

Q4) a) Let A = $\{(a,b): a,b \in R\} \cup \{\{0\}\}$

1) **(5 pt.)** Show that A be considered as a basis for some topology $\mathcal{T}$ on R?

2) **(2 pt.)** Explain Why $\mathcal{T}$ is not the standard topology on R?

3) **(3 pt.)** Compare with $R_{standard}$ and $\mathcal{T}$.

b) Let f, g: (R, $\tau_{styandard}$) → (R; $\tau_{standars}$) be continuous functions . Prove or disprove:

1) **(3 pt.)** the set $\{x \in R : f(x) \leq g(x)\}$ is closed.

2) **(3 pt.)** the function h : (R, $\tau_{styandard}$)→ (R, $\tau_{standars}$), defined as $h(x) := \max\{ f(x), g(x)\}$ for $x \in R$ is continuous.

c) **(5 pt.)** Show that (X, τ) is Hausdorff space **iff** the set $D=\{(x, x) \in X \times X: x \in X\}$ is closed in X×X.

مع تمنياتنا بالتوفيق

STATE OF PALESTINE
Faculty of Applied Science
Time: Two Hours
Date: 31/5/2018 "second semester 2017-2018"

جامعة الأقصى

AL-AQSA UNIVERSITY
Department of Mathematics
Abstract Algebra 1 (Math 3313)
Final Exam

اسم الطالب/ة:	Q1	Q2	Q3	Q4	Q5	Q6	Total
الرقم الأكاديمي:	10	10	10	10	11	9	60
مدرس المساق: د. أحمد محمود الأشقر							

Answer all the following questions: ملاحظة: الامتحان 6 أسئلة ، في 6 صفحات

(Q1) (i) Mark each of the following **True** (√) or **False** (×) : (5 marks)

( ) (1) Let $a,b \in G$ (G is a group). If $|a|=12$ and $|b|=35$ then $\langle a\rangle \oplus \langle b\rangle$ is cyclic group.

( ) (2) If $G=(\mathbb{R}^*,\cdot)$ and $H=(\mathbb{R}^+,\cdot)$, then the index $|G:H|=\infty$.

( ) (3) Let $\phi : G \to \overline{G}$ be a group homomorphism. If $g \in G$ with finite order, then $|\phi(g)|$ divides $|g|$.

( ) (4) The permutation $\alpha=\begin{bmatrix}1&2&3&4&5&6&7&8&9\\9&4&5&2&1&8&7&6&3\end{bmatrix}$ is even .

( ) (5) The relation $\phi:\mathbb{Z}_{12}\to\mathbb{Z}_{10}$ defined by $\phi(x)=(3x)\bmod 10$ is a homomorphism .

(ii) **Circle** the correct answer for each of the following: (5 marks)

(1) Let $a \in G$ (G is a group). Then $|a|=|a^2|$ iff $|a|$ is ...

(a) even (b) odd (c) ∞ (d) (b or c)

(2) The **order** of the factor group $(\mathbb{Z}_{20}\oplus U(20))/\langle(4,7)\rangle$ is ...

(a) 160 (b) 20 (c) 8 (d) 4

(3) The **maximum order** of any element in A_8 is ...

(a) 15 (b) 18 (c) 7 (d) 12

(4) The group $\mathbb{Z}_{15}\oplus\mathbb{Z}_4$ isomorphic to ...

(a) $\mathbb{Z}_{30}\oplus\mathbb{Z}_2$ (b) $\mathbb{Z}_{12}\oplus\mathbb{Z}_5$ (c) $\mathbb{Z}_{60}$ (d) $\mathbb{Z}_6\oplus\mathbb{Z}_{10}$

(5) The **order** of the element $12+\langle 9\rangle$ in the factor group $\mathbb{Z}_{36}/\langle 9\rangle$ is ...

(a) 36 (b) 9 (c) 4 (d) 3

(Q2) (a) Let $G = \left\{ \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R},\ b \neq 0 \right\}$ under matrix multiplication, and

$$H = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} : x \in \mathbb{R} \right\}.$$

(i) Show that H is a subgroup of G . (4 marks)

(ii) Is $H \lhd G$? Justify your answer? (3 marks)

(b) Let $\beta \in S_7$ and suppose $\beta^3 = (4316752)$. Find β in disjoint cycle form . (3 marks)

(Q3)(a) Let $\phi : G \to \overline{G}$ be a homomorphism, prove that $\phi(a) = \phi(b)$ iff $a\,\text{Ker}\phi = b\,\text{Ker}\phi$.

(3 marks)

(b) Define $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_{30}$ by $\phi(x) = 10x \pmod{30}$ $\forall x \in \mathbb{Z}_{12}$

(i) Show that ϕ is a homomorphism .

(3 marks)

(ii) Find Ker ϕ .

(1.5 marks)

(iii) Find $\phi^{-1}(20)$.

(1.5 marks)

(iv) Choose: $\mathbb{Z}_{12}/\text{Ker}\,\phi \approx \ldots\ldots$ $(\mathbb{Z}_2,\ \mathbb{Z}_3,\ \mathbb{Z}_4,\ \mathbb{Z}_{10})$

(1 mark)

(Q4)(a) Let G and H be finite cyclic groups. Show that $G \oplus H$ is cyclic iff $\gcd(|G|,|H|) = 1$.

(4 marks)

(b) Find two groups G and H such that $G \not\approx H$, but $\text{Aut}(G) \approx \text{Aut}(H)$. (3 marks)

(c) Let G be a group with the following property: " If $a,b,c \in G$ and $ab = ca \Rightarrow b = c$ ".

Prove that G is Abelian. (3 marks)

(**Q5**)(a) Determine the number of elements of order 15 and the number of cyclic subgroups of order 15 in $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$. (3 marks)

(b)(i) Express $U(784)$ as an external direct product of groups of the form $\mathbb{Z}_k$. (3 marks)

"Hint: $784 = 16 \cdot 49$ *"*

(ii) What is the largest order for elements in $U(784)$. (2 marks)

(c) Prove that A_n is normal in S_n. (3 marks)

(Q6)(a) State and prove **Lagrage's Theorem**. (3 marks)

(b) Suppose G is finite Group of order n and $\gcd(n,k)=1$. If $g \in G$ and $g^k = e$, prove that $g = e$. (3 marks)

(c) Suppose that G is a non-Abelian group of order p^3 (where p is prime), and $Z(G) \neq \{e\}$. Prove that $|Z(G)| = p$. (3 marks)

انتهت الأسئلة مع أمنياتنا للجميع بالنجاح والتوفيق

AL-AQSA UNIVERSITY
Department of mathematics
Date: 2 /06/2018 (Second semester 2017-2018) جامعة الأقصى

Calculus 1(Math 1411)
Final Exam
Time: 2 Hours

الاسم: الرقم الأكاديمي: مدرس المساق: د. سحر السمك

ملاحظة / عدد الصفحات (6) و عدد الأسئلة (6)

Q1) a) Find $\frac{dy}{dx}$ for the following: **[13 marks]**

1) $y=\left(5x^{2}+\sin 2x\right)^{-\frac{3}{2}}$ (2 marks)

2) $x^2 \cot(5x+10y) = y \cos x$. (3 marks)

2) $y=\int_{x^2}^{\cos x}\sqrt{t+1}\quad dt$ (3 marks)

b) Find the value (s) of **c** that satisfying the **mean value theorem** for derivative

if $f(x)=x+\frac{1}{x}$ on $\left[\frac{1}{2}, 2\right]$ (3 marks)

c) Find $\delta > 0$ that show $\lim_{x\to 3} x^2 + 1 = 10$. "Take $\varepsilon = 1$". (2 marks)

Q2) a) find the following limits: [8 marks]

1) $\lim_{x\to 1^-} \frac{\lceil x+1 \rceil}{x}$. (2 marks)

2) $\lim_{x\to -5^+} \frac{5-x}{x^2 - 25}$. (2 marks)

3) $\lim_{x\to 0} \frac{\sec 5x}{x^2 \csc^2 x}$ (2 marks)

b) Sketch the graph of $f(x) = -(\sqrt{x+1}) - 2$ (2 marks)

Q3) a) Evaluate the following integrals: [12 marks]

1) $\int \frac{2}{\sqrt[3]{x^5}} + \tan^2 x \quad dx$ (2 marks)

2) $\int \frac{3x}{\sqrt{x^2+7}} \quad dx$ (2 marks)

3) $\int \cos^3 x \sin^{16} x \quad dx$ (2 marks)

4) $\int 3x^5 \sqrt{x^3 - 2}\, dx$ (2 marks)

b) Find **asymptotes** and **sketch** the graph of the function $y = \frac{x^2 - 3}{x - 2}$ (4 marks)

Q4) Let $f(x) = x^4 - 4x^3 + 5$ [9 marks]

(a) List the intervals on which $f(x)$ is increasing and decreasing , then find the local extreme values of $f(x)$. (3 marks)

(b) List the intervals on which $f(x)$ is concave up and concave down, then find the inflection points (if eisxt). (3 marks)

(c) Sketch the graph of $f(x)$. (3marks)

(Q5) **[8 marks]**

(a) Find the area of the region enclosed by the parabola $y = 4 - x^2$ and the line $y = 2 - x$.

(4 marks)

(b) Find the **volume** of the solid generated by revolving the region in the first quadrant bounded by the curve $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the x-axis. (4 marks)

Q6) Choose the correct answer: **[10 marks]**

1) The domain of the function $f(x)=\dfrac{1}{\sqrt{x^2-16}}$ is :

a) (-4 , 4) b) $(-\infty,-4)\cup(4,\infty)$ c) $\Re-\{-4,4\}$ d) $\Re$

2) The range of the function $f(x)=\sqrt{16-x^2}=$

a) (-4 , 4) b) [0, 4] c) $[0,\infty)$ d) $\Re$

3) The period of the function $f(x)=\tan(\frac{x}{3})$ is :

a) $\dfrac{\pi}{3}$ b) 3π c) $\dfrac{2\pi}{3}$ d) 2π

4) The vertical asymptote(s) of the function $f(x)=\dfrac{x^2-2x-3}{x^2-1}$ is (are) :

a) $x=1$ b) $x=-1$ c) $x=\pm 1$ d) there are no vertical asymp.

5) $\sin^2 x=$

a) $\dfrac{1-\cos 2x}{2}$ b) $\dfrac{1+\cos 2x}{2}$ c) $1-\cos^2 x$ d) a and c

6) If $f(x)=\cos^2(3-x)$, then $f'(0)=$

a) $-2\cos 3$ b) $2\sin 3\cos 3$ c) $6\sin 3\cos 3$ d) $-2\sin 3$

7) The solution set of the inequality $\dfrac{6}{x+3}\geq 1$ is

(a) $(-\infty,3]$ (b) $[3,\infty)$ (c) $(-3,3]$ (d) $[-3,3]$

8) $\int_{-\pi}^{\pi} \sqrt[3]{x}+\sin^5 x \quad dx$

a) 2π b) π c) 0 d) 1

9) $\lim_{x\to 0}\tan\left(\dfrac{\sin x}{x}-1\right)=$

a) 1 b) 0 c) $\sqrt{3}$ d) the limit does not exist

10) At $x=0$, the function $f(x)=x^{\frac{1}{3}}$

(a) has an inflection point (b) has a vertical tangent (c) is continuous (d) all of them is true.

انتهت الأسئلة ، مع تمنياتي للجميع بالنجاح و التوفيق

AL-AQSA UNIVERSITY

Department of mathematics

Complex Analysis (Math 3311)

Final Exam

Date: 27-5-2018

Time: Two Hours

Answer all of the following questions:

Q1) a) (6 pt.) Show that the function $f(z)=\sinh x \cos y + i \cosh x \sin y$ is differentiable and find $f'(z)$ as a function of z. What about the function

$g(z)= \sinh x \cos y - i \cosh x \sin y$? explain .

b) (6 pt.) Find all complex solutions z for the equation $\sin(z) + 3\cos(z) = 1$

Q2) a) (6 pt.) Show that the function $f(z) = (\bar{z})^2 + z$ is not analytic.

b) **(2 pt.)** Compute $\cos(\pi + i)$.

(c) **(6 pt.)** Find all $z \in C$ such that $\exp(2z) = \sqrt{3} - i$

Q3) a) **(6 pt.)** Show that $|\sinh z|^2 = \sinh^2 x + \sin^2 y$. (4 marks)

b) **(6 pt.)** Find the principal value of the following complex power $(-i)^i$

Q4) a) **(6 pt.)** Suppose z_0 is any constant complex number interior to any simple closed curve contour C. Show that for a positive integer n,

$$\oint_C \frac{dz}{(z-z_0)^n} = \begin{cases} 2\pi i, & n=1 \\ 0 & n>1 \end{cases}$$

b) **(6 pt.)** Let C be the curve given by a half-circle from 1 to -1 (positively oriented) followed by a straight line from -1 to 1. Compute the integral

$$\int_C (z + 1 + \bar{z})dz$$

(c) **(6 pt.)** Compute the values of the following complex line integrals.

(a) $\displaystyle\int_{|z|=4} \frac{z^3+z+1}{(z-1)^3}\,dz$

(b) $\displaystyle\int_{|z|=4} \frac{z^3+z+1}{z^3-6z^2+5z}\,dz$

d) **(4 pt.)** Without evaluating the integral, show that

$$\left|\int_C \frac{dz}{z^2-1}\right| \le \frac{\pi}{3}$$

where C is the arc of the circle $|z| = 2$ from 2 to 2i that lies in the first quadrant

Q5) a) **(5 pt.)** Prove that if f(z) and $f(\bar{z})$ are both analytic in a domain D, then f(z) is constant in D.

b) **(5 pt.)** Prove that an analytic function $f : C \to C$ satisfying $|f(z)| \leq 4$ for any $z \in C$ must be constant

انتهت الأسئلة مع أمنياتنا للجميع بالنجاح والتوفيق

Time: 2 hours

...........Final Exam. For the 2^{nd} sem. of the year 2017/2018

Course: *Introduction to L.P. and O.R.*

AL-AQSA UNIVERSITY

Science Block

Math. Dep. ......

Name:-

Solve the following questions:-

Q(1) a- If a L.P.P. has a feasible solution then show that it also has a basic feasible solution (10marks)

b- For the following L.P.P.

$Maximize\ \ Z = X_1 + 2X_2 + 4X_3$

Subject to the constrants

$X_1 + 3X_2 + 4X_3 = 7$

$X_1 + 3X_2 + 5X_3 = 7$

$X_1, X_2, X_3 \geq 0$

(1) Reduce the feasible solution (1 , 2 , 0) to a basic feasible solution. (10marks)

(2) What is the number of the basic solutions ? (4marks)

Q(2) Use Simplex method to solve the following L.P.P.

(10 marks)

$$Max\ Z = 4X_1 + 3X_2 - X_3$$

Subject to the constrants

$$2X_1 + 3X_2 - 5X_3 \leq -30$$

$$X_2, X_3 \geq 0$$

$$X_1\ \textit{Unrestricted in sign}$$

Q(3) Using the Big M-method solve the following L.P.P. (*For only one iteration*)

$$Maximize \quad Z = 2X_1 + X_2 + 3X_3$$

Subject to the constrants

$$X_1 + X_2 + 2X_3 \leq 5$$

$$2X_1 + 3X_2 + 4X_3 = 12$$

$$X_1, X_2, X_3 \geq 0$$

(12 marks)

Q(4) We have 4-jobs each of which has to go through 3-machines in the order $M_1M_2M_3$ (12 marks)

Processing time (in hours) is given below

Job/Machine	M1	M2	M3
A	12	6	5
B	8	7	8
C	7	2	10
D	10	5	9

Determine a sequence that minimizes the total elapsed time and hence find the total elapsed time , Idle time for each machine.

Q(5) A project consists of 14 activities A,B,C,....M,N the notation $X < Y$ means (12 marks) that the activity X must be finished before Y can begin, with this notation

A<D,H ; B<E ; C < I,F ; D <G ; H, L<M ; E,I<L ; E,F<K ; E,I <J ; G,J,K<N.

The time in days of completion of each activity is as follows:-

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Time	13	8	8	8	3	3	18	8	13	3	8	18	3	23

(i) Draw the project network

(ii) Determine the earliest and latest starting and completion times of activities.

(iii) Identify the critical path

GOOD LUCK

STATE OF PALESTINE

AL-AQSA UNIVERSITY

دولة فلسطين

جامعة الأقصى

Al-Aqsa University

Faculty of Applied Science Department of Mathematics

Linear Algebra II (MATH 2317) Instructor: Dr. Mohammad Hamoda

Date: 24/05/2018 Final Exam Time: Two Hours There are 6 questions in 6 pages

Read the questions carefully. Be neat and organized.

Question(1):- Mark True or False: [15 marks]

1. [ ] The column vectors of an $n \times n$ matrix A span $\mathbb{R}^n$ iff the orthogonal complement of the row space of A is $\{0\}$.
2. [ ] If A^2 is an invertible matrix, then $\lambda = 0$ is an eigenvalue of A.
3. [ ] The set of vectors $\{(2-3i, i),\ (3+2i, -1)\}$ is a basis for the Euclidean complex vector space $\mathbb{C}^2$.
4. [ ] Let V be the vector space of all complex-valued functions, then the vectors $f = 3 + 3i\cos 2x$, $g = \sin^2 x + i\cos^2 x$ and $h = \cos^2 x - i\sin^2 x$ are linearly dependent.
5. [ ] If v_1, v_2 and v_3 come from different eigenspaces of A, then it's impossible to express v_3 as a linear combination of v_1 and v_2.
6. [ ] If A is diagonalizable matrix, then there is a unique matrix P such that $P^{-1}AP$ is a diagonal matrix.
7. [ ] Between any $n-$dimensional vector space and $\mathbb{R}^n$, there is exactly one isomorphism.
8. [ ] The row space of a matrix is isomorphic to its column space.
9. [ ] Any vector space is isomorphic to one of its proper subspaces.
10. [ ] An $n \times n$ matrix A is diagonalizable iff there is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A.

Question(2):- [12 marks]

(I) Find bases for the eigenspaces of the matrix:

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$ [5 marks]

(II) Find a vector u in $\mathbb{R}^4$ that is orthogonal to $v = (1, 0, 0, 0)$ and $w = (0, 0, 0, 1)$, and makes equal angles with $b = (0, 1, 0, 0)$ and $c = (0, 0, 1, 0)$. [4 marks]

(III) Give a definition of: Orthogonally diagonalizable matrix – Quadratic form.
[3 marks]

Question(3):- [11 marks]

(I) Find the geometric and algebraic multiplicities of the matrix:

$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$. Is A diagonalizable? Explain. [6 marks]

(II) Prove that if λ is an eigenvalue of A, x is a corresponding eigenvector and k is a scalar, then $\lambda - k$ is an eigenvalue of $A - kI$, and x is a corresponding eigenvector.

[5 marks]

Question(4):- [11 marks]

(I) Use the method of diagonalization to solve the system:

$y'_1 = y_1 + 4y_2$, $y'_2 = 2y_1 + 3y_2$. [6 marks]

(II) Let $u = (u_1, u_2)$ and $v = (v_1, v_2) \in \mathbb{C}^2$. Is $< u \ , \ v >= u_1\overline{v_1}$ defins a complex inner product on $\mathbb{C}^2$? Clarify your answer. [5 marks]

Question(5):- [10 marks]

(I) Classify the quadratic form $x_1^2 - x_2^2$ as positive definite, positive semidefinite, negative definite, negative semidefinite, or indefinite. [5 marks]

(II) Express the quadratic form $(c_1x_1 + c_2x_2 + \cdots + c_nx_n)^2$ in the matrix notation $x^\top A\, x$ where A is symmetric. [5 marks]

Question(6):- [11 marks]

(I) Let P_2 be the set of all polynomials of degree 2 and P_3 be the set of all polynomials of degree 3. Show that $f: P_2 \longrightarrow P_3$ given by

$a_0 + a_1x + a_2x^2 \longmapsto a_0x + \frac{a_1}{2}x^2 + \frac{a_2}{3}x^3$ is a homomorphism. Does f an isomorphism? Explain. [6 marks]

(II) Let V , W be any two vector spaces and let $f: V \longrightarrow W$ be a homomorphism, suppose that $f(v_1) = w_1$, $f(v_2) = w_2$, $\cdots$, $f(v_n) = w_n$ for some vectors $w_1, w_2, \cdots, w_n$ of W. If $\{v_1, v_2, \cdots, v_n\}$ is linearly independent. Does the set $\{w_1, w_2, \cdots, w_n\}$ linearly independent? Clarify your answer. [5 marks]

Good Luck

Al –AQSA University
Dep. of Math
Date: 29 /05/2018

Numerical Analysis(MATH3322)
Final Exam
Time : 2 hour

Do all the following questions using a five decimal-place mantissa:

Question (1) **[15 Marks]**

A) Find the order of convergence of Newton's method

A) Starting with the definition of forward difference show that $D = \frac{1}{h} Ln\,(1 + \Delta)$

Question (2) **[25 Marks]**

Given the evenly spaced data of (x_i, f_i) values

x_i	1	2	3	4	5	6
f_i	0.5	0.6	0.2	0.8	1.2	1.5

a) Use the Newton-Gregory forward polynomial of degree two to estimate $f(2)$, $f'(2)$.

b) Use the central-difference formula to estimate $f'(3)$.

c) Use the central-difference formula to estimate $f''(4)$.

d) Use the data of the table to find $\int_1^6 f(x)dx$ using Simpson's $\frac{1}{3}$ - rule.

Question (3) **[15 Marks]**

A) Find he local truncation error of Trapezoidal rule

Question (4) **[15 Marks]**

A) Use the modified Newton's Method to solve the non-linear system.

$$\begin{aligned} x + 2y &= 3 \\ 2x^2 + y^2 &= 5 \end{aligned}$$

Start with $x^{(0)} = (-0.9\ ,\ 2.25)^T$ for two iterations.

b) Use Newton's method on the equation $x^3 = N$ to drive the algorithm

$$x_{i+1} = \frac{1}{3}\left(2x_i + \frac{N}{x_i^2}\right)$$

دولــة فلسطيـن

جامعة الأقصى

كليـــــــة العلـــــوم

قســـــم الرياضيات

الفصل الثاني 2017 –2018 م الفترة الثانية	الاختبار النهائي في مساق (أساسيات رياضيات) (MATH 1211)	التاريـــــــخ: 26 /5/ 2018 م الزمـــــــن : ساعتان.

اسم الطالب/ة: ....رقم الطالب/ة:........................ التخصص:........................ الدرجة: 60

....

أجب/ي عن الأسئلة التالية

السؤال الأول: (كل فقرة أربعة درجات)

1) أوجد/ي حل المعادلة $\frac{3(5x-2)}{2}+\frac{1}{3}=\frac{x-4}{6}$

....

2) أوجد/ي حل المعادلة $x^{1/2}+3-4x^{-1/2}=0$

....

3) ما هو الحد الذي سيضاف إلي التعبير الجبري x^2-4kx للحصول علي مربع كامل.

....

4) أوجد/ي قيمة k التي تجعل للمعادلة جدرين حقيقين متساويين $2x^2+kx+8=0$.

....

5) أوجد/ي حل المعادلة التالية بطريقة **اكمال المربع** $x^2+5x-2=0$

....

السؤال الثاني : (كل فقرة أربعة درجات)

1) أوجد/ي ناتج $(321)_5 \times (13)_5 =$ ....

....

2) أوجد/ي حل المعادلة **حسابيا**

$$5x+3y=4$$
$$x+3y=-4$$

....

3) أوجد/ي حل المعادلة **بطريقة التحليل** $x^2-x-6=0$

....

4) أوجد/ي حل المعادلة $\frac{x}{x-2}=\frac{2}{3}$

....

5) أوجد/ي حل المعادلة $x-6\sqrt{x}+8=0$

....

السؤال الثالث : (كل فقرة خمسة درجات)

1)**أكتب/ي جدول الصدق للقضية التالية وبين/ي ما نوعها** $\left[(A \wedge B) \leftrightarrow (C \rightarrow \bar{C})\right] \vee \bar{B}$

2) **أوجد/ي حل المعادلة** $\sqrt{2x-1} = 1 + \sqrt{x-1}$

3) برهن/ي باستعمال قانون الاستنتاج الرياضي

$$4 + 8 + 12 + \ldots + 4n = 2n(n+1) \quad , \quad \forall n \in N$$

4) أوجد/ي ناتج $= (13)_5 \div (2321)_5$

انتهت الأسئلة - مع التمنيات بالتوفيق

STATE OF PALESTINE
Faculty of Applied Science
Time: Two Hours
Date: 27/5/2018 "second semester 2017-2018"

جامعة الأقصى

AL-AQSA UNIVERSITY
Department of Mathematics
Complex Analysis 1 (Math 3311)
Final Exam

اسم الطالبة:	Q1	Q2	Q3	Q4	Q5	Q6	Total
الرقم الأكاديمي:	10	11	9	8	12	10	60
مدرس المساق: د. أحمد محمود الأشقر							

Answer all the following questions:

ملاحظة: الامتحان 6 أسئلة ، في 6 صفحات

(Q1) (a) Prove that $|z| \le |\text{Re}\, z| + |\text{Im}\, z| \le \sqrt{2}\,|z|$ for any complex number z (5 marks)

(b) Show that $\lim_{z\to\infty} f(z) = w_0$ iff $\lim_{z\to 0} f\left(\frac{1}{z}\right) = w_0$ (3 marks)

(c) Use part (b) to find $\lim_{z\to\infty} \frac{3z^2 - i}{iz^2 + 3}$ (2 marks)

(Q2)(a) Sketch the set $G=\{z\in\mathbb{C}:|z-4|<|z|\}$ in the complex plane, and find its closure.

(3 marks)

(b) For any complex number $z=x+iy$ show that $|\sin z|^2=\sin^2 x+\sinh^2 y$,

and use this to show that $|\sinh y|\leq|\sin z|\leq\cosh y$.

(5 marks)

(c) Find the set of all accumulation points of the set $K=\left\{(-1)^n\dfrac{(1+i)(n-1)}{n}:n\in\mathbb{N}\right\}$.

(3 marks)

(Q3)(a) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic, then find a harmonic conjugate $v(x, y)$. (4 marks)

(b) Show that the function $f(z) = e^{-\theta}\cos(\ln r) + ie^{-\theta}\sin(\ln r)$ is differentiable in the domain $(r > 0,\ 0 < \theta < 2\pi)$. and find $f'(z)$. (5 marks)

(Q4) (a) Find all complex number z such that $e^{2z-3} = 1+\sqrt{3}i$. (3 marks)

(b) Show that $\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$. Then find all solutions of the equation $\tan z = 3i$.

(5 marks)

(Q5)(a) Evaluate $\int_C \frac{z+2}{z}\, dz$, where C is the semicircle $z = 2e^{i\theta}$,$(\pi \le \theta \le 2\pi)$. (4 marks)

(b) Without evaluating the integral, show that $\left| \int_C \frac{\operatorname{Log} z}{z^3}\, dz \right| < 2\pi \left(\frac{\pi + \ln R}{R^2} \right)$,

where C denotes the circle $|z| = R \;\; (R > 1)$, taken counterclockwise . (4 marks)

(c) Let a function $f(z)$ be analytic in a domain D. If $|f(z)|$ is constant in D, prove that $f(z)$ must be constant in D. (4 marks)

(Q6)(a) State the **Cauchy integral formula** . (2 marks)

(b) Evaluate the following integrals:

(1) $\int_C \frac{dz}{(z^2+9)^2}\, dz$, where C denotes the positively oriented circle $|z+2i|=3$. (4 marks)

(2) $\int_C \frac{e^z \cos z}{(z+3i)(z-i)}\, dz$. where C denotes the positively oriented circle $|z|=2$. (4 marks)

انتهت الأسئلة مع أمنياتنا للجميع بالنجاح والتوفيق

STATE OF PALESTINE

AL-AQSA UNIVERSITY

دولـة فلسطيـن

جامعـة الأقصـى

كلية العلوم التطبيقية

جامعة الأقصى

الامتحان النهائي في مادة تفاضل وتكامل ١ — الزمن: ساعتان

اسم الطالبة:..

شعبة-يوم :..

التخصص ....

Answer the following questions:

1. Find the area of the region between the graphs $y = 4 - x^2$ and $y = -x + 2$ where $-2 \le x \le 3$

2. Find the horizontal asymptotes of $f(x) = \frac{3x}{\sqrt{2x^2 - 5}}$

3. Find $\lim\limits_{x \to 0} \dfrac{x - \tan 7x}{2x}$

4. Find the domain and range of $f(x) = \dfrac{5}{1 - \sqrt{x}}$

5. Find $\displaystyle\int \frac{\tan^2(x)\sec^2(x)\,dx}{\left(1 + \tan^3(x)\right)^5}$

6. Find $\dfrac{dy}{dx}$:

(i) $y = \cos^4\left(\csc^2(3x)\right)$

$(ii) \qquad y = \int_{\sqrt{x}}^{x^2} \frac{dt}{1+t^2}$

$(iii) \qquad \int_{\sqrt{x}}^{x^3} \frac{dt}{t+\sin(t)}$

7. Find $\lim\limits_{x \to -5} \sqrt{1-3x}$, then find $\delta \succ 0$ that works for $\varepsilon = 0.5$

8. Find increasing, decreasing intervals , local extreme values, [illegible] for the curve $f(x) = x^4 - 4x^3$

9. Find the average value of $f(x) = \sec^2(x)$ on $[0, \frac{\pi}{4}]$

10. Find $\int x^3 \sqrt{7+x^2}\,dx$

11. Find the volume of the solid generated by revoling the region bounded by the curves $y=\sqrt{x}$, $y=2$ and $x=0$: (i) about $y=2$

(ii) about $x=5$

12. Find the asymptotes of the graph of $f(x)=\dfrac{3x^2}{x+1}$ and sketch the graph of $f(x)$.

13. Find: $\displaystyle\int_0^{\frac{\pi}{10}} \cos^2(5x)\,dx$

14. Use the Max-Min inequality to find a lower bound for the value of the integral

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1+3\csc(x)}\, dx$$

15. Find the value of c that satisfies the Mean Value Theorem (for derivatives) for $f(x) = x + \dfrac{1}{x}$ on $[1, 2]$

انتهت الأسئلة

دولـــة فلسطيـن
جامعة الأقصى

كليــــــة العلــــــوم
قســـــم الرياضيات

الفصل الثاني ٢٠١٨	الاختبار النهائي في مساق (تفاضل وتكامل ٢)	التاريـــــخ: ٢٠١٨/٦/٢م
محاضر المساق: قسم الرياضيات	رقم المقرر (MATH1412)	الزمــــــن : ســاعتان .

ملاحظات : عدد الصفحات 6. عدد الأسئلة: 6 اسم الطالب/ ة ....الرقم الاكاديمي......................

Q1) Choose the correct answer: [10 marks]

1) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both divergent, then $\sum_{n=1}^{\infty} a_n + b_n$ is :

a) Convergent b) divergent c) may converge or may diverge. d) none of the above

2) The series $\sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{3^n}$ is

a) alternating series b) geometric series c) power series d) a and b

3) $\cosh^2 x - \sinh^2 x$ equal to

a) 1 b) $\cosh(2x)$ c) e^x d) non of the above

4) If the points (r, θ) *and* $(-r, -\theta)$ lie on the graph of the curve $r = f(\theta)$, then the graph is symmetric about:

a) $X-axis$ b) $Y-axis$ c) the pole(origin) d) all the above

5) The sum of the series $\sum_{n=0}^{\infty} \left(\frac{-1}{9}\right)^n$ is equal to:

a) $\frac{9}{8}$ b) $\frac{1}{8}$ c) $\frac{9}{10}$ d) $\frac{1}{10}$

6) The parabola $x = -2y^2$ has focus at:

a) $(0, -\frac{1}{8})$ b) $(-\frac{1}{8}, 0)$ c) $(0, -8)$ d) $(-8, 0)$

7) The polar coordinates of the center of the circle $r = -8\sin\theta$ is:

a) $(-4, \frac{\pi}{2})$ b) $(-4, -\frac{\pi}{2})$ c) $(4, -\frac{\pi}{3})$ d) $(4, \frac{\pi}{2})$

8) The domain of $y = \log_7 x$ is

a) $(-\infty, \infty)$ b) $(0, \infty)$ c) $[0, \infty)$ d) $\Re - \{0\}$

9) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$

a) diverges b)converges c) absolute converges d) may converge or may diverge

10) The polar equation which equivalent to the Cartesian equation $xy = 1$ is

a) $r^2 + r\sin\theta = 1$ b) $r^2 \sin(2\theta) = 2$ c) $r = 1 + \sin\theta\cos\theta$ d) $r = 1$

Q2) [12 marks]

a) Find $\frac{dy}{dx}$, if $y = \log_3\left[\cos^{-1}(\tanh x)\right] + 7^{\sec x}$ [3 marks]

b) Consider the function $f(x) = x + 2\sqrt{x}$. Find $\left(\frac{df^{-1}}{dx}\right)_{x=8}$ [3 marks]

c) Solve for x :

$\ln(x-2) = 4 + \ln x$ [3 marks]

d) Find the following limit: [3 marks]

$$\lim_{x \to 0^+} \left[\cos(\sqrt{x})\right]^{\frac{1}{x}}$$

Q3) [8 marks]

a) Find the center, foci, vertices, and asymptotes of the conic section: [4 marks]

$4x^2-9y^2+4x+54y+44=0$.

b)_ Find the radius and the interval of convergence (abs. conv., cond. Conv.) of the power series:

$\sum_{n=1}^{\infty}\frac{(x+1)^n}{\sqrt{n}}$. [4 marks]

Q4) [9 marks]

a) Find **<u>Taylor series about zero (a = 0)</u>** generated by the function $f(x)=\cos(2x+\pi)$. [3 marks]

b) Find **<u>binomial series</u>** generated by the function $f(x)=\dfrac{1}{\sqrt{x+2}}$. [3 marks]

c) Let $r=\dfrac{8}{2+2\sin\theta}$ be an equation of conic section with one focus at the origin: [3 marks]

a) Identify the conic section.

b) Find the directrix that corresponds to the focus at the origin.

c) Sketch it's graph.

Q5) [12 marks]

a) let $r = 2 + 4\sin\theta$ [4 marks]

1) Sketch the graph of the curve.

2) Find the area inside the curve.

b) Test the convergence for each of the following: [8 marks]

1) $\sum_{n=1}^{\infty} \frac{4}{(3n+1)(3n-1)}$ (2 marks)

2) $\sum_{n=1}^{\infty} \frac{e^{n}}{(2n)!}$ (2 marks)

3) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n+5}{n}\right)^n$ (2 marks)

4) $\int_1^e \frac{1}{x\ln(x)}\, dx$ (2 marks)

Q6) Find the following integral [9 marks]

1) $\int x^4 \log_3 x \quad dx$ (3 marks)

2) $\int_{\ln 2}^{\ln 3} \frac{e^x}{\sqrt{1-e^{2x}}} \quad dx$ (3 marks)

3) $\int \frac{x^{\ln x} . \ln x}{x} \quad dx$ (3 marks)

جامعة الأقصى

كلية العلوم	امتحان نهاية الفصل الثاني	3/6/2018
قسم الرياضيات	لمادة : رياضيات تطبيقية 1	الزمن : ساعتان

الاسم:- ....

أجب عن الأسئلة التالية:-

السؤال الأول:- إذا كان

$$\vec{A} = Sin\, u\ \hat{i} + Cos\, u\ \hat{j} + 5u\,\hat{k}$$

$$\vec{B} = Cos\, u\ \hat{i} - Sin\, u\ \hat{j} + 5\hat{k}$$

$$\vec{C} = 3\hat{i} + 6\hat{j} - 4\hat{k}$$

أوجد $\frac{d}{dt}\left[\vec{A}\bullet(\vec{B}\times\vec{C})\right]$ (8 درجات)

السؤال الثاني:-

هل مجال القوة

$$\vec{F} = (y^2 z^3 \cos x - 4x^3 z)\hat{i} + 2yz^3 \sin x\ \hat{j} + (3y^2 z^2 \sin x - x^4)\hat{k}$$

تحفظيا وإن كان كذلك أوجد/ي دالة الجهد العددي Φ **(12 درجة)**

السؤال الثالث:-

1- أوجد المشتقة الاتجاهية للدالة العددية $\Phi = 3x^2y - 6y^3z^2$ في الاتجاه $\hat{i} - 2\hat{j} - 2\hat{k}$

(8 درجات)

2- احسبي الشغل المبذول لتحريك جسيم بواسطة القوة $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$
على طول الخط المستقيم من النقطة (4 , 2 , 1) إلى النقطة (6 , 5, 2) **(12 درجات)**

السؤال الرابع:

- هل مجال القوة

$$\vec{F} = (y^2 z^3 \cos x - 4x^3 z)\hat{i} + 2yz^3 \sin x\,\hat{j} + (3y^2 z^2 \sin x - x^4)\hat{k}$$

تحفظيا وإن كان كذلك أوجد/ي دالة الجهد العددي Φ **(12 درجة)**

السؤال الخامس:

1- إذا كان $\vec{A}$ متجه ثابت أثبت أن $\vec{\nabla}(\vec{r}\cdot\vec{A})=\vec{A}$ **(6 درجات)**

2- إذا كان $\vec{\nabla}\times\vec{A}=\vec{0}$ احسب $\vec{\nabla}\cdot(\vec{A}\times\vec{r})$ **(6 درجات)**

3- أثبت أن $\vec{\nabla}|\vec{r}|^{3}=3\vec{r}\, r$ **(6 درجات)**

GOOD LUCK

دولـــة فلسـطيـن

جامعة الأقصى

كليـــــــة العلـــــوم

قســــــم الرياضيات

جامعة الأقصى

26-05-2018

الفصل الثاني 2018 م	الاختبار النهائي في مساق : مدخل الي المنطق ونظرية	التاريــــــخ: 2018/5/26 م
محاضر المساق: أ. د عبدالسلام أبوزايدة	المجموعات	الزمـــــــن : ســاعتان .
	رقم المقرر MATH 1313	

ملاحظات :عدد الصفحات: 3 عدد الأسئلة: 3 اسم الطالب/ة ....

I-True or False:- [10 Marks]

1. φ is an inductive set.
2. PMI is equivalent to PCI.
3. $\{\{4\}\} \in \{1,2,3,\{4\}\}$.
4. If $A \subseteq B$, then $A^c \subseteq B^c$.
5. If A and B are finite sets, then A-B is finite.
6. Every infinite set is uncountable.
7. Every finite set is countable.
8. Every subset of infinite set is finite .
9. 7+5=12 iff 1+3=5.
10. Every denumerable set is countable.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.

II- Prove the following :

1. Use contradiction to prove: $\varphi - A = \varphi$. [5 Marks]

2. If $f : A \to B$, then $f \circ I_A = f$. [5 Marks]

3. Define a relation S on Z by: x S y iff $x^2 = y^2$

   i- Prove that S is an equivalence relation. [4 Marks]

   ii. Describe the equivalence classes for S. [3 Marks]

4. Prove that if $f : A \xrightarrow{1-1} B$ and $g : B \xrightarrow{1-1} C$, then $g \circ f : A \to C$ is 1-1 function. [6 Marks]

III- Solve all the following questions:-

1. Show that for any set A and $x \notin A$, then $A \approx A \times \{x\}$ [5 marks]

i- 2. prove : $P(A \cap B) = P(A) \cap P(B)$. [5 Marks]

3. Given a function, $f(x) = x^2$

ii- Find $f^{-1}([1,4))$ [2 Marks]

iii- $f([1,2])$. [2 Marks]

iii. $f((-1,2) \cup (2,3))$. [2 Marks]

4. Use the PMI to prove $2 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$, $\forall n \in N$ [6 Marks]

iv- Prove by any method: if S is finite and $x \notin S$, then $S \cup \{x\}$ is finite. [5 Marks]

انتهت الأسئلة

Final Exam for the 2nd sem.
Course : Selected Topics(مواضيع مختارة)
Date: 24/5/2018
Time: Two Hours

STATE OF PALESTINE
AL-AQSA UNIVERSITY
Faculty of Science
Math Dep

الاسم:.

Solve the following questions:

(Q1)

(i)If G is not connected and $|V|=21$ then find the maximum number of edges in G (6 marks)

(iii) How many r-regions do the graph K_{30} has? (4 marks)

(iv) Find a graph homeomorphic to $K_{2,2}$ with minimum number of edges (2 marks)

(Q2) (i) Let T be a binary tree of height h then show that T has between $h+1$ and $2^{h+1}-1$ vertices (6 marks)

(ii) If T is a full binary tree with 20 internal vertices then find the number of leaves (4 marks)

(iii) Represent the expression $\dfrac{\left(xy^{2}+3x^{4}\right)z}{y+z^{2}}$ by a binary tree (3 marks)

(Q3)

(i) For which m and n does $K_{m,n}$ (6 marks)

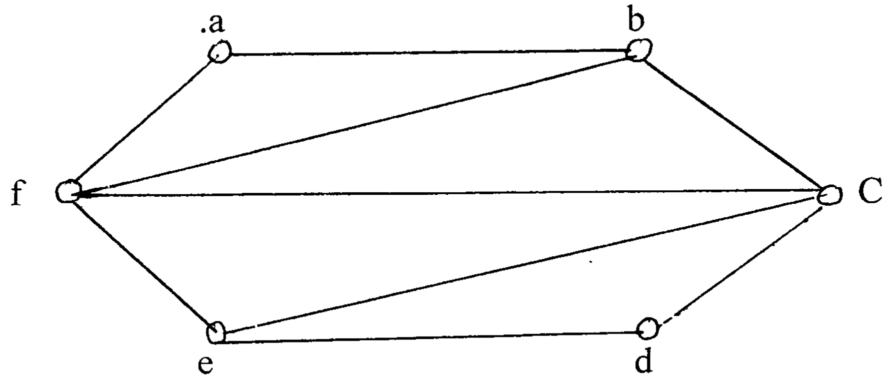
1- Eulerian

2- Hamiltonian

3- Planar

(ii) Let G = (V, E) be a connected planar simple graph with 30 vertices each of degree 4
Into how many regions does a representation of this planar graph splits the plane
(4 marks)

(iii)Conceder the following graph

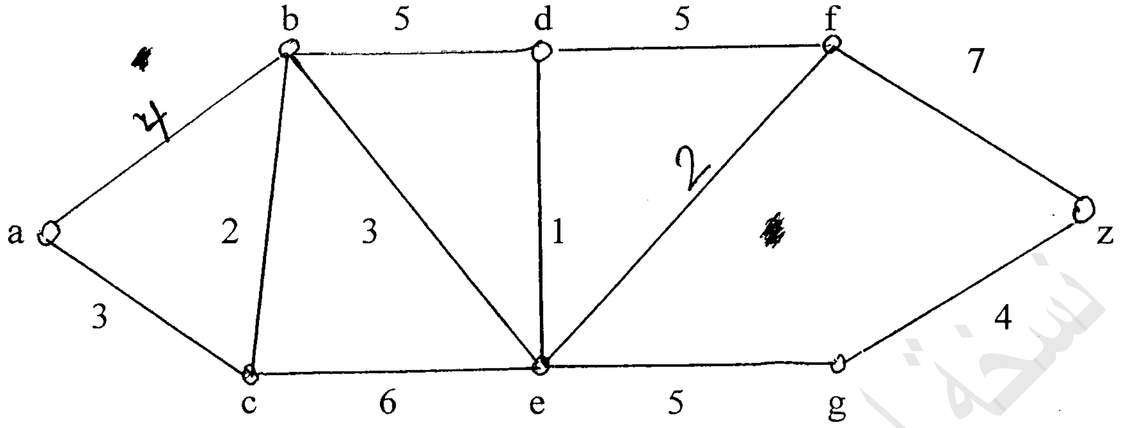


1- Is G has an Euler path? If yes find it (5 marks)

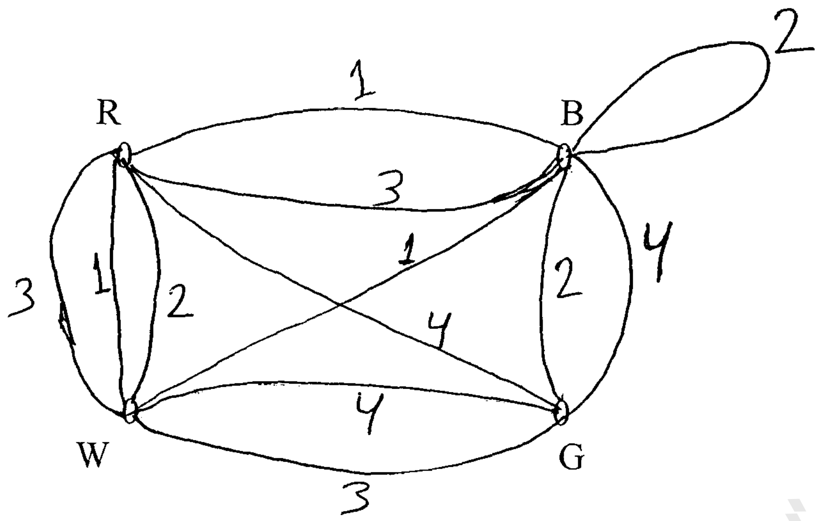
2- Is G a planar graph ? If not why ? (4 marks)

3- Find the adjacency matrix for the graph (2 marks)

(Q4) Find the minimal spanning tree for the following weighted graph (5 marks)



(Q5) Find a solution for the following Instant Insanity puzzle. (7 marks)



(Q6) 1- What are the chromatic numbers for the following? (4 marks)

I – C_{101} (cycle with 101 vertices)

II - $K_{2,25}$

2- Schedule the final exams for the following courses shown in the following Boolean matrix using fewest numbers of periods (8 marks)

	*Math*115	*Math*116	*Math*185	*Math*195	*CS*101	*CS*102	*CS*273	*CS*473
*Math*115	0	0	0	1	1	1	1	0
*Math*116	0	0	1	1	1	1	1	0
*Math*185	0	1	0	0	1	1	1	1
*Math*195	1	1	0	0	0	0	1	1
*CS*101	1	1	1	0	0	1	1	1
*CS*102	1	1	1	0	1	0	1	1
*CS*273	1	1	1	1	1	1	0	1
*CS*473	0	0	1	1	1	1	1	0

🕮 ***GOOD LUCK*** 🕮

دولـــة فلسطيــن | جامعة الأقصى
جامعة الأقصى | قســــم الرياضيات
كليــــــة العلــــــوم

الفصل الثاني 2018 م	الاختبار النهائي في مساق : مواضيع مختارة في الهندسة	التاريـــــخ: 2/6/2018 م
محاضر المساق: أ. د عبدالسلام أبوزايدة	رقم المقرر : MATH 2317	الزمن: ساعتان.

ملاحظات :عدد الصفحات: 3 عدد الأسئلة: 8 اسم الطالب/ة ....

1 – **True or False** [10 Marks]

i. The set of rotations form a commutative group.
ii. The composition of two primitives is a primitive.
iii. Translation fixes a line that is parallel to its vector.
iv. Reflection in the Y-axis maps $(1, 2)$ to $(-1, -2)$.
v. Reflection in the axis passes through the origin with angle 0 fixes the vertical lines .
vi. The set of all similarities of negative ratio forms a group.
vii. Primitive transformation fixes a line pointwise.
viii. Affine transformation preserves the area of the triangle.
ix. Every similarity is an isometry.
x. Similarity of ratio $k = 2$ preserves the surface of the circle .

i.	ii.	iii.	iv.	v.	vi.	vii.	viii.	ix.	x.

2 – **Show that the set of all transformations of the form:**

$$T: \begin{array}{l} x' = x \\ y' = k\,y \end{array} \qquad , k \succ 0$$

Forms a group. [6 Marks]

3 – **Given a transformation T by :**

$$T: \begin{array}{l} x' = 2x + 3y + 1 \\ y' = 6x + 4y + 2 \end{array}$$

Decompose T into primitive transformations. **[10 Marks]**

4 – **Given a transformation T by :** **[10 Marks]**

$$T: \begin{array}{l} x' = \frac{1}{2}x + \frac{\sqrt{3}}{2}y \\ y' = \frac{\sqrt{3}}{2}x - \frac{1}{2}y \end{array}$$

i. **Show that T is a reflection.**

ii. **Find the axis of T.**

iii. **Find the image of the line** $y = \sqrt{3}\,x$ **.**

5 – **What is the displacement that maps the line $y = 2x + 1$ to the line $y = 2x$.** [6 Marks]

6 – **Find the equation of the radial similarity with center (a, b) and ratio k.** [6 Marks]

7 – **Show that the similarity with ratio k maps the circle with area A to the circle with area k^2 A.**
[6 Marks]

8 – **Prove that the translation preserves the collinearity.** [6 Marks]

انتهت الأسئلة

نظرية الاحتمالات		جامعة الأقصى – غزة
الامتحان النهائي		قسم الرياضيات
المحاضر/ د. عصام عاطف داود	الفصل الدراسي الثاني 2017-2018	التاريخ: 2018/5/21 (الفترة الثانية)

السؤال الأول: أجب عن الأسئلة الآتية حسب المطلوب: **(6 درجات)**

1. (درجتان) بكم طريقة مختلفة يمكن اختيار 3 كرات حمراء و 4 كرات صفراء من صندوق يحتوي على 5 كرات حمراء و 6 كرات صفراء؟
2. (درجتان) كم عدد التبديلات المختلفة الموجودة في أحرف كلمة "**BOOK**"؟
3. (درجتان) ماهو معامل $x^2y^3z^3$ في $(x+y+z)^8$؟

السؤال الثاني: أثبت كلاً مما يلي: **(10 درجات)**

1. (3 درجات) $p(\phi)=0$.
2. (3 درجات) اذا كان $p(A)=a$ و $p(B)=b$ فإن $p(A|B)\geq \dfrac{a+b-1}{b}$.
3. (4 درجات) $\sum_{r=0}^{n} r\binom{n}{r}=n2^{n-1}$

السؤال الثالث: أجب حسب المطلوب: **(8 درجات)**

1. (4 درجات) إذا كان $X_1, X_2, \ldots, X_n$ متغيرات عشوائية مستقلة تتبع توزيع بواسون (Poisson) بنفس المعلمة λ. أوجد التوزيع الاحتمالي للمتغير Y حيث $Y=X_1+X_2+\ldots+X_n$
2. (4 درجات) إذا كان X يتبع التوزيع الأسي (Exponential) بمعلمة θ. أوجد دالة الكثافة الاحتمالية (pdf) للمتغير Y حيث $Y=\ln(X)$.

السؤال الرابع: أجب حسب المطلوب: **(15 درجة)**

1. إذا كان $X\sim N(5,25)$
   - أ. (3 درجات) أوجد $p(-10<X<10)$
   - ب. (3 درجات) أوجد قيمة a حيث $p(X>a)=0.90$
2. (3 درجات) إذا كانت ماكنة معينة تحتاج إلى تصليح بمعدل مرة واحدة كل 3 سنوات، ما احتمال أن تعمل الماكنة لمدة على الأقل 5 سنوات دون الحاجة إلى تصليح؟
3. (3 درجات) إذا كان احتمال شخص ما يصدق إشاعة = 0.75، فما احتمال أن الشخص الثامن الذي يسمع الإشاعة هو الشخص الخامس الذي يصدقها؟
4. (3 درجات) تقدم 12 شخص لوظيفة ما، 8 أشخاص منهم مؤهلين، اذا تم اختيار 5 أشخاص منهم بشكل عشوائي للمقابلة، فما احتمال أن يكون منهم 2 فقط مؤهلين إذا كان السحب بإرجاع؟

السؤال الخامس: إذا كان $f(x,y)$ تعطى من خلال الجدول التالي: **(9 درجات)**

		X		
		-1	0	1
Y	-1	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$
	1	$\frac{1}{6}$	0	$\frac{1}{6}$

1. (6 درجات) أوجد $Cov(X,Y)$
2. (3 درجات) هل X و Y متغيران مستقلان؟ أجب مع التفسير.

السؤال السادس: **(12 درجات)**

إذا كان

$$f(x,y)=e^{-x-y} \quad , x>0, \ y>0$$

أوجد $Var(W)$ حيث $W=3X+4Y-5$

السؤال الإضافي: BONUS **(5 درجات)**

1. (2.5 درجة) إذا كان $X \sim N(5,25)$ فأوجد b حيث $p(|X|>b)=0.10$
2. (2.5 درجة) وضح لماذا لايوجد متغير عشوائي له الدالة المولدة للعزوم الآتية: $M_X(t)=\frac{t}{1-t}$.

مع تمنياتي للجميع بالنجاح والتفوق

Some Formulas:

$$X \sim Poisson\,(\lambda) \Rightarrow f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x = 0,1,2,\ldots$$

$$E(X) = \lambda, \quad Var(X) = \lambda, \quad M_X(t) = e^{\lambda(e^t - 1)}$$

$$X \sim Exponential\,(\theta) \Rightarrow f(x) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, x > 0$$

$$E(X) = \theta, \quad Var(X) = \theta^2, \quad M_X(t) = \frac{1}{(1-\theta t)}$$

$$X \sim binomial\,(n,\theta) \Rightarrow f(x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}, x = 0,1,2,\ldots,n$$

$$E(X) = n\theta, \quad Var(X) = n\theta(1-\theta), \quad M_X(t) = (\theta e^t + (1-\theta))^n$$

$$X \sim Gamma\,(\alpha,\beta) \Rightarrow f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{\frac{-x}{\beta}}, x > 0$$

$$E(X) = \alpha\beta, \quad Var(X) = \alpha\beta^2, \quad M_X(t) = \frac{1}{(1-\beta t)^{\alpha}}$$

$$X \sim negative\,binomial\,(k,\theta) \Rightarrow f(x) = \binom{x-1}{k-1} \theta^k (1-\theta)^{x-k}, x = k, k+1,\ldots$$

$$X \sim geometric(\theta) \Rightarrow f(x) = \theta(1-\theta)^{x-1}, x = 1,2,\ldots$$

$$X \sim hypergeometric \Rightarrow f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

$$\sum_{i=0}^{n} \binom{n}{i} X^i Y^{n-i} = (X+Y)^n$$

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx$$

Table III: Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4988
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Also, for z = 4.0, 5.0, and 6.0, the probabilities are 0.49997, 0.4999997, and 0.499999999.